Homework 12T

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit, all your answers should be carefully justified. For this homework, you do NOT need to defined sample space for Q1 and Q2, but you should still explicitly define your sample space for Q3.

Please make sure to strictly follow our collaboration policy as clarified on piazza @1716 @1750 @1882.

1. **[10 pts] I'm like TT, just like TT (Part 2)**

   As a die-hard member of the ONCE fandom, Richard is listening to his TWICE playlist when Jasmine presents two claims to him, asking Richard if they are True (T) or False (F). However, Richard isn't paying attention because he's too engrossed jamming out to TWICE, and he sings out loud "I'm like TT, just like TT!" Thinking he was answering her question, Jasmine marks both claims as True. Help Jasmine check this by proving or disproving the following claims (Note: if disproving, only disproving via valid counterexample will receive credit):

   (a) Markov's inequality holds for all random variables

   (b) If Z is a random variable that takes on values at most 20 with $\mathbf{E}[Z] = 10$ and $\Pr[Z < 0] \neq 0$, Markov's inequality can still be used to show that $\Pr[Z \leq 5] \leq \frac{2}{3}$

2. **[10 pts] ONE IN 212 TRILLION**

   It is a well-known fact that, on average, 212 trillion people view each music video released by TWICE. Yuyang, the biggest fanboy of them all, has been given the ultimate task of directing their next music video.

   (a) Yuyang wants to make the next music video the best one yet in the hopes that his crush will finally notice him. He decides that least 265 trillion people need to view his video to have the best chance of this happening. Using Markov's inequality, help Yuyang find an upper bound on the probability that his music video will be viewed by at least 265 trillion people.

   (b) Satisfied with this upper bound, Yuyang wants to present his findings to the founder, JYP. For his presentation, help him define a distribution of music video viewership where the probability that a music video was viewed by at least 265 trillion people is exactly equal to the upper bound you provided in part (a).

   (c) During his meeting with the founder, Yuyang learns that no music video released by

TWICE has ever been viewed by fewer than 53 trillion people! Happy that his favorite kpop group was doing so well, he then tries to use this information to improve his upper bound on the probability that a TWICE music video was viewed by at least 265 trillion people. Can he improve this bound? If so, explain how and provide a new distribution as you did in part (b). If not, explain why the new information does not help.

(d) It is also a well-known fact that the standard deviation of the number of people who view each TWICE music video is equal to 2 trillion people. Using Chebyshev's inequality, help Yuyang find an upper bound on the probability that a music video was viewed by at least 265 trillion people.

3. **[10 pts] DON'T CALL ME AGAIN (unless you have TWICE tickets)**

Tien and Lucy are desperately trying to find tickets to a socially-distanced TWICE concert. So, they decide to call TWICE fans until someone gives them their tickets. Each of the fans that Tien and Lucy call have an independent $\frac{1}{6}$ chance of giving them tickets. Tien and Lucy will stop calling TWICE fans either when a fan gives them tickets, or after they call 5 fans, whichever happens first. If each call takes exactly 15 minutes, what is the expectation and variance of the number of minutes they spend calling fans?