CIS 160

Homework 11H

Assigned: November 12, 2020

Due: 9:00 a.m. EST, November 17, 2020

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit, all your answers should be carefully justified; in particular, please make sure to explicitly define your sample space for any probability question unless otherwise specified.

Please make sure to strictly follow our collaboration policy as clarified on piazza @1716 @1750 @1882.

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1. **[25 pts] Beans and Cheesecake? Natto Good Idea!**

   Tien is notorious for liking objectively bad food and today, he is making bean cheesecakes for his fellow CIS 160 TAs. He has 30 natto beans and 30 red beans to add as toppings for his 30 cheesecakes. Each of the natto and red beans are added to a cheesecake selected uniformly at random, and each cheesecake can have any number of beans. Tien is hoping that at the end, each cheesecake will have a good mix of red beans and natto beans. Help him out by calculating the expected number of cheesecakes with at least one bean of each kind.

2. **[30 pts] I... DECLARE... BANKRUPTCY!!!**

   The Flagstaff team is feeling pretty stressed this week, and so they decided to drive to Vegas and let off some steam at the casinos. Matthew decides to try his luck with the slot machine. However, Matthew just keeps losing and losing and losing (he's really struggling), so Matthew decides to use his coding skills and hack into the machine. Unfortunately, his coding skills are not very good, so instead of rigging the game to win, all he did was corrupt the final result screen. The slot machine is supposed to print out a 8-letter string "BANKRUPT" every time Matthew loses on the slot machine, but now, the slot machine has been corrupted so that it continuously prints out a stream of characters where each character is chosen uniformly at random from the 26 capital letters. Of course, Kara still wants to see Matthew lose all of his money and go bankrupt (Kara: "it's fun when Matthew is sad!"), so how many letters must be printed so the expected number of occurrences of the string "BANKRUPT" is exactly 1?

3. **[15 pts] Building Beantopia With the Trusty Toothpick**

   Yuyang is world-building for his next fantasy novel. Luckily for Yuyang, he has a big jar of beans and toothpicks and uses each bean to represent a kingdom in his world and toothpicks connecting beans to represent an alliance between kingdoms. In Yuyang's world, each kingdom has at least $d \geq 0$ allies. Prove that if there are no alliance-triangles (i.e. cycles of exactly three kingdoms), Yuyang needs at least $2d$ beans to represent his world.