

Homework 10t

Due: 9:00 am EST, November 5, 2020

This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in []. To receive full credit, all your answers should be carefully justified; in particular, please make sure to explicitly define your sample space for any probability question unless otherwise specified. Please see Piazza for the updated collaboration policy.

Also, please remember to double check that you have submitted the correct version of your homework onto Gradescope by re-downloading it.

1. [10 pts] Conveyor Conversion Cost Cuts

At Bethany and Jared's renowned ice cream factory, there is a legendary conveyor belt system that can deliver ice cream between $n \geq 1$ offices. Each office has some number (possibly 0) of conveyor belts connecting it to other offices, although there is at most 1 conveyor belt between any 2 offices. If there is a conveyor belt between 2 offices, ice cream can be sent in either direction between the offices.

Jared wants to cut down on costs by making the conveyor belts one-way. However, Bethany worries that if an employee crafts a delicious new flavor and wants to send the first batch around the factory so everyone can try it, then that employee won't be able to get their batch back on the new one-way conveyor belt system. Is it possible to convert the two-way belts to one-way ones in such a way that no ice cream would be able to return to its creator? Prove your answer.

2. [10 pts] No Funny Name Here, You Just Gotta Prove It

Prove that if Y is a random variable which only takes on values that are non-negative integers, then:

$$\mathbf{E}[Y] = \sum_{i=0}^{\infty} \Pr[Y > i]$$

3. [10 pts] Lots of Lactose, Lots of Luck

After a successful few weeks, Kara and Matthew have collected a lot of milk jugs from their milk farm. With their excess milk, they invent a new game, 160-jack, which has rules as follows:

Before the game starts, Kara tosses a fair coin (one side of the coin is labeled 1 and the other side is labeled with a 2) and rolls a fair 160-sided die (the sides are labeled from 1 to 160), but she doesn't show Matthew the results.

Now, Matthew must pay 20 milk jugs to play. Matthew then decides if he wants to see the result of the coin toss or the die roll first. If the result of the object he chooses is a 1 (with

either the coin toss or die roll), he wins 160 milk jugs and moves to the next round; if it is any other number, he must pay Kara 32 milk jugs and does not move to the next round.

In the second round, Kara reveals the result of the other object. For instance, if Matthew chose the die in the first round, he will now see the result of the coin toss, and vice versa. If the result of the second object is a 1, Matthew wins 1600 milk jugs. If it is any other number, he loses and must pay Kara 32 milk jugs.

Matthew is left with the following options:

- (a) Decide to not to play the game.
- (b) Play the game and look at the coin first.
- (c) Play the game and look at the die first.

Find the expected value of Matthew's milk jug winnings of each of these options and suggest which option Matthew should play with to get the highest milk jug winnings in expectation.