

CIS 160 — Mathematical Foundations of Computer Science

Homework Assignment 8T

Assigned: October 19, 2021

Due: 8:30 AM ET, October 21, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, October 21, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:

- 5 points will be deducted from your homework if you do not use the provided L^AT_EX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in L^AT_EX.

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. If multiple solutions are given, only the first one will be graded. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may organize into collaboration teams of up to 3 current students. For each homework assignment, you can only be in one team and must list all team members on your homework submission using the provided L^AT_EX template, whether or not you specifically spoke with them. You may have different teams for different assignments. Collaboration must be strictly limited to discussion, and solutions must be written separately. For the complete collaboration policy, please consult the announcement on Piazza. Violations may seriously affect your grade in the course.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. [6 pts] Prepare for trouble...

It's PokIshaan week at CIS160 headquarters! Team Rocket is back at it again with their nefarious schemes, and it's up to the gang to put a stop to them!

Team Rocket's first dastardly act was to steal all the PokIshaan from the Center! Originally, the trainer had some integer number of PokIshaan between 0 and 8. Each PokIshaan is distinct, but was one of two forms, either ChaRichards or Ria Sharmanders. One day, Krish was able to spot two of the lost ChaRichards. The trainer tells Krish that the chance that any two of her lost PokIshaan, uniformly chosen at random, both being ChaRichards, is exactly $1/2$. How many PokIshaan did the trainer lose, and how many of each type?

2. [8 pts] ...and make it double!

Oh no! The PokIshaan team finds itself in a conundrum as Team Rocket devises another scheme against them. Fortunately, Jayrachi's current hibernation period is almost at its end, after which he will awaken for 7 days with the ability to grant wishes. However, due to his limited powers, Jayrachi can only grant 9 wishes throughout the week, but there are 35 wishers — 5 members of the PokIshaan team making a wish on each day of the week. None of the wishes are overlapping.

To make his grants as fair as possible, Jayrachi randomly chooses 9 of the 35 possible wishes to fulfill. Each wish is equally likely to be picked, and Jayrachi doesn't pick any of the wishes twice. What is the probability that Jayrachi will fulfill at least one wish on every day of the week?

3. [8 pts] Quinnlava's Secret TM

Quinnlava has discovered a breakthrough in Pokémon training. His new technique maximizes training efficiency, and he wants to share his knowledge with other Pokémon trainers. He goes to a local Pokémon trainer meeting, where he counts $n \geq 2$ people in attendance, including himself.

Ria Sharmander, the host of the meeting, claims that each of the n people in the meeting is friends with at least $\frac{n}{2}$ of the other Pokémon trainers. Note that each of these friendships is mutual (that is, if Person A is friends with Person B, then Person B is friends with Person A). Quinnlava then tells all of his friends about the new technique; these friends tell all of their friends, and so on.

Prove that all n Pokémon trainers will eventually learn Quinnlava's technique.

4. [8 pts] Trainer Tip: An Aspear Berry can be used by a Pokémon to thaw itself out

Calamity! Trainer Helen doesn't have enough berries to make her famous pie for her annual

Halloween Pokémon trainer party! In the field by the training gym, there are Aspear berries and Belue berries, and Helen needs at least 7 of one kind of berry to make her pie. Helen takes her trusty Elyssa Pikachou to scour the field for berries in order for Helen to continue her mission to be the very best (party host) like no one ever was. Every time Elyssa Pikachou spots some berries, she'll hand it to Helen, and with each berry spotting, she has an equal and independent probability of discovering an Aspear berry or a Belue berry. As soon as the Pokémon team has found 7 of one kind of berry, they'll head home to make the pie. What is the probability that Elyssa Pikachou will make exactly 12 berry spottings before they go home? You make assume that on each spotting, Elyssa Pikachou collects either an Aspear berry or a Belue berry.