

CIS 160 — Mathematical Foundations of Computer Science

Homework Assignment 8H

Assigned: October 21, 2021

Due: 8:30 AM ET, October 26, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, October 26, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:

- 5 points will be deducted from your homework if you do not use the provided L^AT_EX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in L^AT_EX.

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. If multiple solutions are given, only the first one will be graded. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may organize into collaboration teams of up to 3 current students. For each homework assignment, you can only be in one team and must list all team members on your homework submission using the provided L^AT_EX template, whether or not you specifically spoke with them. You may have different teams for different assignments. Collaboration must be strictly limited to discussion, and solutions must be written separately. For the complete collaboration policy, please consult the announcement on Piazza. Violations may seriously affect your grade in the course.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. [12 pts] Lindaquil x Cindawill

Lindaquil and Cindawill are as close as two Cyndaquils can be. However, they have been feeling a little bit lonely, so they adventure from Penn to the Johto region to find some new friends! In Professor Elm's lab, they find 7^n distinguishable Cyndaquils, where $n \in \mathbb{N}$. Excited by all these new potential companions, Lindaquil asks each Cyndaquil how many of the other Cyndaquils (not including Lindaquil and Cindawill) they are *not* friends with. Lindaquil sums up all of these answers and gets an answer that is at most 7^k , where $n \leq k < 2n$ and $n, k \in \mathbb{N}$. Assuming that each friendship is symmetric, prove that there exists a Cyndaquil who has at least $7^n - 7^{k-n}$ friends!

2. [8 pts] Mischievous Molt-Krish

Ar-Taki-uno, Olivia Zha-pdos, and Molt-Krish were meeting together for their annual Kanto committee meeting. When the three legendary birds are all together, they have the power to create routes between any of the n towns in the Kanto region. These routes are bidirectional, and between any two towns there can only be at most one route.

However, Molt-Krish is feeling particularly fiery this year, and decides that he wants to prank the entire Kanto region by making each route go in only one direction. That is, for any two towns i and j with a route between them, the route will only allow trainers to go from i to j or j to i , but not both, and it will never change direction. Given that there are trainers living in every town, Molt-Krish claims that no matter what plans he's given, he can cleverly direct the traffic to prevent any trainers from returning to their home town once they've left. Is Molt-Krish's claim possible? Justify your answer. You should provide a strategy for achieving his goal and prove that it must work, or prove that no such strategy exists.

3. [8 pts] RAJIVGIGAS (Credit to @634 on Piazza)

Rajivgigas is enraged to have been awoken from his slumber. The world is depending on you to solve the following problem to calm him down! Let A and B be two events in the same probability space and for any event C , let \bar{C} denote its complement. Derive the following formula:

$$\Pr[(A \cap \bar{B}) \cup (\bar{A} \cap B)] = \Pr[A] + \Pr[B] - 2\Pr[A \cap B]$$

4. [12 pts] This story is bogus because there is no way a pseudo-legendary has never won a battle

Ryan the TyRyranitar is stranded in Towne with no way to get to the PokIshaan tournament. Just as Ryan is about to accept his fate as a sad PokIshaan who has never won in his life, Joseph LEEvee appears out of nowhere and offers him his used Honda Accord, which will help Ryan

get to the tournament. Ryan gladly takes the offer but to his disappointment, he discovers that the Accord does not work according to plan. Each time Ryan steps on the gas pedal, the car will take him 1 km closer to the tournament with probability p , and the car won't move at all with probability $1 - p$.

- (a) What is the probability that after pressing the gas pedal twice, Ryan will be 1 km closer to the PokIshaan tournament than he was when he started?
- (b) What is the probability that after pressing the gas pedal three times, Ryan will be 2 km closer to the PokIshaan tournament than he was when he started?
- (c) Given that after pressing the gas pedal three times, Ryan has moved 2 km closer to the PokIshaan tournament, what is the probability that after pressing the pedal the first time, Ryan ended up 1 km closer to the tournament?

5. [14 pts] I wanna be the very best

To culminate the week, all of the PokIshaan organize a round-robin fight to test their powers! To determine the strongest of them all, every PokIshaan will fight every other PokIshaan exactly once. Fights will only involve 2 PokIshaans at a time and will end with one winner and one loser (a fight cannot end in a tie).

Prove that if there are a total of 2^n PokIshaans, where n is a non-negative integer, we can find $n + 1$ PokIshaans who can be listed in a row such that each PokIshaan in the row has beaten every other PokIshaan that is listed to the right of them.

For example, Tientacool, PiKrishu, BulbaSara, and Rialu is an acceptable list of length 4 if Tientacool has beaten PiKrishu, BulbaSara, and Rialu; PiKrishu has beaten BulbaSara and Rialu; and BulbaSara has beaten Rialu.

6. [16 pts] Today's Top Story: Wild Mewkil Decimates Young PokIshaan Geniuses

After the PokIshaan tournament, it looks as if the strongest PokIshaan has been decided. But wait! A wild Mewkil appeared! Mewkil decides to challenge a line of n PokIshaans to a battle of the ages. Using Mewkil's psychic abilities, PokIshaans are either confused or focused, and initially, some PokIshaans start off confused. Mewkil can only knock out PokIshaans that are already confused, and when a PokIshaan is knocked out, the adjacent PokIshaans will be turned focused if they are confused, confused if they are focused, and still knocked out if they are knocked out. Any PokIshaan that is knocked out will remain knocked out, and the only operation that Mewkil can perform is knocking out confused PokIshaans. For instance, with a starting configuration of

“F”, “C”, “C”, “F”

If Mewkil knocks out the second PokIshaan from the left, the configuration becomes

“C”, “KO”, “F”, “F”

If Mewkil knocks out the only PokIshaan that is confused now, the configuration becomes

“KO”, “KO”, “F”, “F”

Prove that Mewkil can knock out all n PokIshaans if and only if the number of PokIshaans that are initially confused is *odd*.