

CIS 160 — Mathematical Foundations of Computer Science

Homework Assignment 7T

Assigned: October 12, 2021

Due: 8:30 AM ET, October 19, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, October 19, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:

- 5 points will be deducted from your homework if you do not use the provided \LaTeX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in \LaTeX .

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. If multiple solutions are given, only the first one will be graded. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may not collaborate with anyone via any means.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources. **If you use the multiplication rule on a question in this homework, you must explicitly cite the multiplication rule.**

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. [14 pts] Even in football, Justin Fields makes buckets

Justin Fields is preparing for his upcoming game by throwing nine distinct footballs into four distinct buckets. Each football is equally likely to land in any of the buckets, and because Justin Fields is so talented, we are sure that he won't miss, and each football will end up in one of the buckets.

Because his head coach has silly rules, Justin Fields will be ready for his game only if none of the buckets are empty at the end of practice. Otherwise, he will be forced to try again next week. What is the probability that Justin Fields is ready this week?

2. [16 pts] Prove these Cover-2 inequalities

After finally completing his practice task, it's time for the game! This week Justin Fields has a big game against the Packers. Because the defense knows how amazing and goated and wonderful Justin Fields is, they know they cannot stop him with traditional defenses. Instead on each third down, they throw a complicated math inequality at him which he must prove. If he can prove the inequality, then he will score a touchdown, otherwise he will get sacked! Help Justin Fields prove the following inequalities, so he can score a touchdown and kick start his Hall of Fame career! P.S. GO BUCKSSSSS!!!!!!!!!!!!!!!!!!!!!!

Let A and B be two events. For each of these inequalities, answer the following:

- (i) Prove the inequality.
- (ii) Under what conditions for A and B are these inequalities satisfied as equations?

A mathematical derivation is required for each part. (i.e. do not cite the union bound given in recitation for parts (i), and for (ii), please provide a condition for equality based on the relationship of A and B rather than giving an example of sets which satisfy the claim. You do **not** need to prove the claim you provide)

- (a) $\Pr[A] + \Pr[B] \geq \Pr[A \cup B] \geq \max\{\Pr[A], \Pr[B]\}$
- (b) $\min\{\Pr[A], \Pr[B]\} \geq \Pr[A \cap B] \geq \Pr[A] + \Pr[B] - 1$