# CIS 160 — Mathematical Foundations of Computer Science
# Homework Assignment 4T

**Assigned:** September 21, 2021 **Due:** 8:30 AM ET, September 23, 2021

---

This homework is due electronically on Gradescope at 8:30 AM ET, September 23, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

**A. Standard Deductions:**
- 5 points will be deducted from your homework if you do not use the provided LaTeX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in LaTeX.

**B. Solutions:** Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. If multiple solutions are given, only the first one will be graded. *Solutions must be given in closed form (as defined on Piazza).*

**C. Collaboration:** You may not collaborate with anyone via any means.

**D. Citations:** All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources. **If you use the multiplication rule on a question in this homework, you must explicitly cite the multiplication rule.**

**E. Outside Resources:** Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

**F. Late Policy:** We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

**1. [20 pts] Favorite Season?**

Answer each of the following questions. You don't need to show your work.

(a) There are 140 TAs that hate summer and are counting the days until Fall. On campus, there are 21 identical trees with fall colors that the TAs want to photograph. Unfortunately, the distinguishable TAs love Fall so much that they each could photograph more than one tree, and they also refuse to share any trees with each other. How many different ways can the trees be photographed? You may assume that the order in which the trees are photographed does not matter.

(b) If instead, the 140 TAs and 21 trees are all distinguishable, how many different ways can the trees be photographed?

(c) **True** or **False**? In the above question, if neither the TAs nor the trees are distinguishable, there is only 1 way in which the trees can be photographed.

(d) With 21 identical copies of a tree she photographed, Helen decides to make a pattern of pictures. She creates a $21 \times 21$ grid to place the photos on. How many patterns can she make such that no two photos are in the same row or in the same column of the grid?

(e) If the tree photos are distinguishable, how many ways are there for Helen to place the photos on the $21 \times 21$ grid now, such that no two photos are in the same row or in the same column of the grid?

(f) Helen then took another 21 photos of 3 distinct trees, each of which has leaves of different colors. She took 7 photos of the tree with green leaves, 9 photos of the tree with orange leaves, and 5 photos of the tree with red leaves. Photos of the same tree are indistinguishable, but photos of different trees are distinguishable. How many ways can Helen arrange these 21 photos such that no two photos are in the same row or in the same column of the $21 \times 21$ grid?

(g) Helen really liked 11 of the photos that she took, so she taped them up on her wall in a row. She then invited 8 of her TA friends to come to her house and bring frames for her photos. Each of the other TAs has a unique frame, which they will use for exactly one photo, and no two TAs can frame the same photo. Helen also does not want there to be any two adjacent photos that are left without a frame. How many ways can the 8 TAs friends frame Helen's photos?

(h) Helen now wants to hang up 10 more photos on a separate wall. 3 of them are of the tree with green leaves, 3 of them are of the tree with orange leaves, and 4 of them are of the tree with red leaves. How many ways can she order these photos in a row? Assume photos of the same tree are identical.

(i) While Helen was taking photos, Rashmi was outside performing her favorite hobby: collecting leaves. She was ecstatic when she came home with 28 carefully chosen leaves, all of which are distinguishable from one another. She wants to place these new leaves in a page of her special binder, for the memories. The page that she wants to place these leaves in has 2 rows, where each row has 14 spots for the leaves. Of the 28 leaves, 8 are orange leaves and 6 are red leaves, while the remaining leaves are green. Rashmi decides that the 8 orange leaves must be placed in the first row, and the 6 red leaves must be placed in the second row, for aesthetic purposes. How many ways can the 28 leaves be arranged?

(j) Given the beautiful weather, Taki decides to take a selfie on Locust. Since Taki is an extremely talented coder, he created an app that allows him to put filters on his photos. The app has 15 available filters and Taki is determined to apply 2 filters to his selfie. The order in which the filters are applied does not matter and his app allows the user to apply the same filter multiple times. How many different ways can he apply filters to his selfie?

**2. [10 pts] Will Will get his leaf crunching fill?**

Will, an avid leaf cruncher, sees the leaves on Locust are changing color! Unfortunately, he still has his induction homework to complete. Help Will prove the following using induction:

(a) For all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1$$

(b) For all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i(i+1)(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$