CIS 160 — Mathematical Foundations of Computer Science
Homework Assignment 3T

Assigned: September 14, 2021 **Due:** 8:30 AM ET, September 16, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, September 16, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:
- 5 points will be deducted from your homework if you do not use the provided LaTeX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in LaTeX.

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may not collaborate with anyone via any means.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources. **If you use the multiplication rule on a question in this homework, you must explicitly cite the multiplication rule.**

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. [9 pts] Practice Makes Purr-fect

Prove the following.

(a) Let $A = \{n \in \mathbb{N} \mid n = 7k+4, \text{ for some } k \in \mathbb{N}\}$ and $B = \{n \in \mathbb{N} \mid n = 21k + 4, \text{ for some } k \in \mathbb{N}\}$. Prove that $B \subset A$.

(b) Let $A = \{n \in \mathbb{Z} \mid n = 4 - k, \text{ for some } k \in \mathbb{N} \text{ and } 5|k\}$ and $C = \{m \in \mathbb{Z} \mid m = 24 - 5k \text{ for some } k \in \mathbb{N} \text{ and } k \geq 4\}$. Prove that $A = C$.

(c) Let $A = \{n \in \mathbb{N} \mid n = 6k-13, \text{ for some } k \in \mathbb{N}\}$ and $B = \{m \in \mathbb{N} \mid m = 5k + 14, \text{ for some } k \in \mathbb{Z}^+\}$. Prove that $A \neq B$.

2. [6 pts] Did Christian Practice Enough?

Christian the Cat enjoys watching musicals (or has he likes to call them, "meow"-sicals). This past weekend, he went to the local theater to watch Cats the Musical. Un-fur-tunately, Christian the Cat forgot his money to pay for the ticket! The box office clerk told Christian that he could enter for free if he could give a proof for $\sqrt{6} + \sqrt{7} < \sqrt{26}$. Christian responded instantly with the following:

> Squaring both sides of $\sqrt{6} + \sqrt{7} < \sqrt{26}$ gives $13 + 2\sqrt{42} < 26$, which further implies $2\sqrt{42} < 13$. Squaring both sides gives $168 < 169$, which is true.

Help the clerk verify Christian's solution. If it is valid, give a brief justification why (a couple of lines will suffice). If not, explain why the proof is invalid and provide a correct proof.

3. [10 pts] Real Worth Proof Applications

There are kittens galore! Bethany has stumbled through a portal into a world where kittens are everywhere. Of course given how adorable they are, Bethany would like to adopt some to bring back to the 160 TAs. However, Catty Claire stands in her way and will only give the kittens to whomever she deems worthy. To prove herself, Bethany must solve the following set theory proofs.

Help the TAs get the kittens and answer the following questions. We have sets F (furry), T (tabby), C (cuddly), and H (hungry). Note that since this is a magical territory, the sets F, T, C, and H are not the same between each subproblem.

(a) Suppose that F, T, and C are sets with $F \cap T \cap C = \emptyset$. Prove or disprove:

$$|F \cup T \cup C| = |F| + |T| + |C|$$

(b) Prove that if F and $T \setminus C$ are disjoint, then $F \cap T \subseteq C$.

(c) Let F, T, C, and H be arbitrary sets. Prove that

$$(F \cap C) \cup (T \cap H) \subseteq (F \cup T) \cap (C \cup H)$$

(d) Let F and T be arbitrary sets. Prove that if $F \neq \emptyset$ and $F \times T = \emptyset$, then $T = \emptyset$.

(e) Let F and T be arbitrary sets. Prove or disprove:

$$\mathcal{P}(F) \cup \mathcal{P}(T) = \mathcal{P}(F \cup T)$$

4. [5 pts] Timmy Tammy Turnt To Treats

Jasmine's cats Timmy and Tammy are turning one this September! To celebrate, Jasmine wants to buy them some special treats. She researches n distinct treat delivery companies. Each company has 2 types of treats (mini cupcakes vs a custom cake). Jasmine doesn't want to buy 2 things from the same company, but she wants to make sure she orders at least one treat total for her cats. Note that she doesn't have to buy from every company. In how many ways can she do this?