

CIS 160 — Mathematical Foundations of Computer Science

Homework Assignment 1H

Assigned: September 7, 2021

Due: 8:30 AM ET, September 14, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, September 14, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:

- 5 points will be deducted from your homework if you do not use the provided \LaTeX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in \LaTeX .

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may not collaborate with anyone via any means.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources. **If you use the multiplication rule on a question in this homework, you must explicitly cite the multiplication rule.**

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. [9 pts] Silly Goose! Sets are for 160 students!

Give answers to the following questions. You do not have to show your work for this question.

- (a) Write the following sets explicitly, i.e. list the members of these sets.
- $\{x \mid x \text{ is a square of an integer, } x \text{ is even, and } x < 220\}$
 - $\{x \mid x \text{ is an integer such that } x^2 = 37\}$
 - $\{x \mid x \text{ is a real number such that } x^2 - 120 = 1\}$
 - $\{x \mid 3x \text{ is a positive integer less than 57 and } 7 \mid x\}$
- (b) Use the set builder notation to give a nontrivial description of each of these sets (nontrivial means that your solution should not simply enumerate every element).
- $\{8, 11, 16, 22, 24, 32, 33, 40, 44\}$
 - $\{4, 19, 44, 79, 124\}$
 - $\{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$
- (c) What is the cardinality of each of the following sets?
- $\{a, \{a\}\}$
 - $\{\{p, m\}\}$
 - $\{d, \{d\}, \{d, \{d\}\}\}$
- (d) Determine whether each of the following is true or false.
- $\emptyset \subseteq \{x\}$
 - $\emptyset \in \{x\}$
 - $x \subseteq \{x\}$
 - $\{x\} \in \{x\}$
 - $\{x\} \subseteq \{x\}$
 - $\{x\} \in \{\{x\}\}$
- (e) What is the power set of $\{x, y, z\}$, where x , y , and z are distinct elements? What is the cardinality of the powerset?
- (f) Find two sets A and B such that $A \in B$ and $A \subseteq B$.

2. [7 pts] Gaggle Gobbles Grub Game

Helen, Ishaan, Jasmine, Krish, and Linda are taking a morning walk to a park in hopes of feeding the geese. A park RA lines them up in alphabetical order and tells them the following: the first two people must flip a coin to decide who gets to feed the geese first. The winner of the match can then feed the geese and continue on their morning walk, while the loser must stay to play the next person in line. The person who wins the next match gets to feed the geese second, while the loser must again stay to play the next person in line. This process continues until there is only 1 TA left, who feeds the goose last. In how many orders can the 5 TAs feed the geese?

3. [7 pts] Ryan('s) Gosling(s)

Honk honk! Ryan the goose has recently given birth to n little goslings, where n is a positive integer. Ryan wants his goslings to stay in fashion, so he decides to dye their feathers with 28 different colors. To decide how to color his goslings, he makes them float next to each other in a row of n consecutive goslings. He decides that he'd like the colors to be symmetric about the middle of the n goslings. For example, if $n = 4$, then Ryan might dye his goslings Red-Yellow-Yellow-Red; if $n = 5$, he could choose Orange-Blue-Green-Blue-Orange.

Unfortunately, he's still feeling the after effects of giving birth, so he needs your help to do the calculations. How many different ways are there to color the row of n goslings with 28 colors such that the colors are symmetric about the middle?

4. [7 pts] Do geese dap with their wings or their feet?

There are $n \geq 2$ young and ambitious geese competing in the 2021 Tokyo Olympics. All the participating geese are told to meet in a grand hall before the opening. Glistening with excitement, the geese proceed to dap each other up. To clarify, each goose can dap up 0 to $n - 1$ geese and a dap is a mutual act between two parties. Of course, it is done with gloves to prevent spread of COVID-19. Given that there is at least one goose who has not dapped up everyone in the grand hall, what is the maximum number of geese who could have dapped up everyone? Explain your answer.

5. [7 pts] The G in ROYGBIV stands for Goose

When walking along Locust Walk, Tien comes across a rare species of geese called Rainbow Geese. As the name suggests, Rainbow Geese are colored red, orange, yellow, green, blue, and violet (because indigo isn't real). There are 10 red geese, 15 green geese, and 40 violet geese that Tien comes across. All geese of the same color are indistinguishable from one another. Eager to show his roommate, Tien wants to bring some geese (at least 1) back to his apartment. He is

strong enough to carry all 65 geese back to his apartment. How many different ways can Tien bring the geese back to his apartment?

6. [6 pts] What if... we became goose farmers? Jkjk... unless?

Weilin has decided to be a goose farmer and have her geese perform for the visitors of the farm. She wants to arrange the geese in a circle for the start of the performance. She wants to use at least one gander (a male goose) and at least one dame (a female goose), all geese of the same gender are distinguishable; additionally, for each goose, she wants both of its neighbors to be the same gender. If there are 75 Ganders in her final goose circle, how many geese did she use overall?

7. [5 pts] The Great Glorious Glamorous T.A. Gaggle-off

Ohoho! Taki has spotted a gaggle of 1,963 glamorous goslings gregariously gabbing inside of a Towne lecture hall! Elyssa, a die-hard geese fanatic, stumbles upon this fantastic sight and wants to claim the gorgeous goslings for herself, so she challenges Taki to a glorious game for the goslings. If Elyssa wins, she can keep the goslings, and if Taki wins, Elyssa will walk away with no goslings.

The game is as follows: The 1,963 goslings are labeled consecutively from 1 to 1,963 and then placed inside the bio pond. To start, Elyssa will have 0 points. Taki will then fish out a gosling from the pond one at a time and read out the gosling label to Elyssa. Elyssa can then choose to add or subtract that gosling number value from her current score. After all the 1,963 goslings have been fished out from the bio pond, if Elyssa's score is exactly 1, then she wins and keeps the goslings!

Help Elyssa fulfill her gosling dreams and find her a winning strategy. If it is hopeless, explain why she cannot win.

8. [5 pts] Quinn and Nicky are geese btw

Chess geniuses Quinn and Nicky are preparing their 8×8 chess board. Unfortunately, the square colors are completely out of whack! Currently, 4 corner squares are black, while the rest of the board is white. To fix the board, Quinn and Nicky can **peck** at the board. A **peck operation** flips the colors of all the squares in a single row or column. To calibrate the board, the genius geese want to make the entire board black. Is this possible? If so, give an example sequence of operations; if no such sequence exists, explain why.

9. [5 pts] Which weighs heavier: 200 beautiful geese feathers, or what you did to get those feathers?

Will V. and Jay V. walk into a barn. Inside, there is a row of 200 beautiful geese feathers of varying lengths. Feeling artistic, they want to each create the longest feather tapestry in the shape of a V by connecting feathers end to end. The length of a tapestry is defined as the total length of the feathers contained within the tapestry. In order to create the longest tapestry, they do the following: Will V. removes a feather from either end of the original row to be used in his tapestry. Then Jay removes a feather from either end of the remaining row of 199 geese feathers. They continue alternating like this until there are no more geese feathers remaining.

Will V. wants to select feathers so that in the end, the total length of all of his 100 feathers is at least as long as the total length of Jay's 100 feathers. Can you help him find a strategy? If no strategy exists, explain why.

10. [12 pts] Poached Eggs

Yuyang the goose is trying to save his three goose eggs from a sly fox. The fox, being very sly, says to Yuyang that he can have an egg back if he is able to prove or disprove each of the following claims. Help Yuyang save innocent goose lives before it's too late!

- (a) EGG 1: For any positive integer $k > 1$, $4^k - 1$ is never prime.
- (b) EGG 2: Let t be a positive integer. If r is a nonnegative irrational number, then $r^{1/t}$ is irrational.
- (c) EGG 3: For all real numbers x , $\lfloor -x \rfloor = -\lceil x \rceil$.