CIS 160 — Mathematical Foundations of Computer Science
Homework Assignment 13T

Assigned: November 23, 2021   **Due:** 8:30 AM ET, November 30, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, November 30, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:

- 5 points will be deducted from your homework if you do not use the provided LaTeX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in LaTeX.

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. If multiple solutions are given, only the first one will be graded. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may organize into collaboration teams of up to 3 current students. For each homework assignment, you can only be in one team and must list all team members on your homework submission using the provided LaTeX template, whether or not you specifically spoke with them. You may have different teams for different assignments. Collaboration must be strictly limited to discussion, and solutions must be written separately. For the complete collaboration policy, please consult the announcement on Piazza. Violations may seriously affect your grade in the course.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. **[8 pts]** Jealous of Tien's 65 pet geese (from Homework 1H), Andrew decides to go to a farm to get some feathery friends. He brings home n distinguishable turkeys. Since it's Thanksgiving time, Andrew decides to have a reverse-Thanksgiving, and let his turkeys have a feast!

   Andrew cooks k different dishes of food for his turkeys. Each dish is distinct from every other dish. Andrew also wants to make sure that each of the n turkeys get their own dish. Much to his surprise, Andrew learns that turkeys enjoy many different types of food. The number of dishes that each turkey likes is at least as large as the number of turkeys who like any one dish.

   Prove that Andrew can distribute dishes to bird buddies such that each turkey gets a dish that they enjoy, and each dish is eaten by at most one turkey. You may assume that $k \geq n$, and that every turkey likes at least one dish.

2. **[10 pts]** Kyle is on his way to an all-you-can-eat Thanksgiving feast, and is excited to stuff himself with turkey. He's hoping to be able to eat 70 plates of food to make up for not being able to eat 70 dumplings at the CSA dumpling fest. Unfortunately, Kyle has to finish the following equivalence relations problem before he leaves, and he's already running a little late.

   Let R and S be equivalence relations on the same set. Help Kyle prove or disprove the following two statements:

   (a) $R \cup S$ is an equivalence relation.

   (b) $R \cap S$ is an equivalence relation.

3. **[12 pts]** Chef Elyssa Chou is excited to begin preparing her famous annual Thanksgiving dinner, and is looking for ideas for one last item to add to her feast. She decides to use a stream of uppercase letters A – Z, each displayed uniformly at random, to decide which item to add based on its initials. Anusha, a big mashed potatoes fan, is secretly hoping that "MP" will be printed. What is the expected number of letters printed until Chef Chou sees the first instance of "MP" appear on the page?