CIS 160 — Mathematical Foundations of Computer Science
Homework Assignment 12T

Assigned: November 16, 2021 **Due:** 8:30 AM ET, November 18, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, November 18, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:

- 5 points will be deducted from your homework if you do not use the provided LaTeX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in LaTeX.

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. If multiple solutions are given, only the first one will be graded. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may organize into collaboration teams of up to 3 current students. For each homework assignment, you can only be in one team and must list all team members on your homework submission using the provided LaTeX template, whether or not you specifically spoke with them. You may have different teams for different assignments. Collaboration must be strictly limited to discussion, and solutions must be written separately. For the complete collaboration policy, please consult the announcement on Piazza. Violations may seriously affect your grade in the course.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. **[15 pts]** Weilin is trying to figure out if she can win the 160 TA Fantasy Football league. Her team has been facing some hard times (only winning once against Jay) after drafting Free Agent Virgil Green 96th overall who has not played a game this season. She calculates her average score every week is 60 points. You may assume that all scores are nonnegative (assume no player can score negative points for her team).

a. Weilin calculates that she will need to score 90 points in the next week to beat Kyle. Knowing only Weilin's average score, use Markov's inequality to find the best possible upper-bound for the probability that she beats Kyle.

b. Demonstrate that this is the best possible bound by giving a distribution for which this bound holds with equality.

c. Suddenly, Weilin remembers that she never scores less than 45 points in any week of Fantasy Football. How does this allow you to improve your previous bound? As before, show that this is the best possible bound.

d. Now suppose you *further* learn (in addition to the information in (c)) that the standard deviation of Weilin's scores is 2. What is the Chebyshev bound on the probability that Weilin scores at least 90 points?

2. **[15 pts]** After being the only one to lose against Weilin in the 160 TA Fantasy Football league by forgetting to put Christian McCaffrey in his lineup, Jay is pretty sad and decides to punish himself by not leaving his room until he solves a few problems. Help Jay figure out the following problem so he can learn his lesson.

Let Y be a random variable such that $Y = \sum_{i=1}^{n} Y_i$, where each Y_i is a random variable. Prove that if $\mathbb{E}[Y_iY_j] = \mathbb{E}[Y_i]\,\mathbb{E}[Y_j]$, for every pair i, j, such that $1 \leq i < j \leq n$, then

$$\mathrm{Var}[Y] = \sum_{i=1}^{n} \mathrm{Var}[Y_i].$$