CIS 160 — Mathematical Foundations of Computer Science
Homework Assignment 11T

Assigned: November 9, 2021   **Due:** 8:30 AM ET, November 11, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, November 11, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:

- 5 points will be deducted from your homework if you do not use the provided LaTeX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in LaTeX.

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. If multiple solutions are given, only the first one will be graded. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may organize into collaboration teams of up to 3 current students. For each homework assignment, you can only be in one team and must list all team members on your homework submission using the provided LaTeX template, whether or not you specifically spoke with them. You may have different teams for different assignments. Collaboration must be strictly limited to discussion, and solutions must be written separately. For the complete collaboration policy, please consult the announcement on Piazza. Violations may seriously affect your grade in the course.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two 'T' homeworks and two 'H' homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. **[10 pts] Yes, we know, but we needed 22 letters!**

   It's a week before Taylor Swift's new release of her album Red, and Taylor is staring down at her finished re-recorded album in glee. In celebration, Taylor Swift excitedly shouts "Tonight's the night where we forget about the deadlines!" and starts dancing like she's 22. Unfortunately, in her excitement, Taylor accidentally bumps into her computer and completely corrupts her recorded album, so that all of the lyrics are scrambled and indecipherable. Taylor also somehow managed to save and close the software so that she can't simply click "CTRL+Z" and fix her album.

   Luckily for her, one of her most loyal Swifties Joseph Lee has created a program that can un-scramble Taylor's new album. However, Joseph coded the program so that it would only activate given a string containing a specific line of 22 letters from one of Taylor's songs. Unfortunately, Joseph has forgotten which string he set as the password. Unbeknownst to him and Taylor, the correct string is "ONANBALCONYINSUMMERAIR". Taylor immediately begins typing a stream of uppercase letters, where each letter is chosen independently and uniformly at random from the alphabet. What is the fewest number of random letters Taylor would need to enter for the expected number of occurences of "ONANBALCONYINSUMMERAIR" to be at least 1? (Because Joseph is feeling extremely generous, you do NOT need to define a sample space for this question).

2. **[10 pts] This says a lot about how much Brandon values graph theory**

   Shhh! Don't leak the secret, but Jay is actually a huge Taylor Swift fan. Also, definitely DO NOT call him Jaylor Swift. Anyways...

   With the new album Red coming out, Jaylor Swift really wants to get a signed copy. Luckily, Brandon is willing to offer him a free signed copy, but only if Jaylor can prove or disprove the following conjecture about graphs:

   Let G be a simple, undirected graph with at least one edge. Construct another graph G' as follows — for each edge e in G, we create a unique corresponding vertex v_e in G'. Then, for any two vertices v_{e_1} and v_{e_2} in G', there is a corresponding edge $\{v_{e_1}, v_{e_2}\}$ in G' if the edges e_1 and e_2 in G are incident on the same vertex. If G has an Eulerian circuit, then G' has a Hamiltonian cycle.

   Help Jaylor Swift prove or disprove the statement so he can fulfill his "wildest dream" of getting a free signed album!

3. **[10 pts] "Title this boring question." -David**

   Now, Jaylor Swift really wants to get his back signed by Taylor Swift (he has come up with a

plan to shower without wetting his back). Brandon has connections with Taylor Swift's manager and is able to provide Jaylor this lucky opportunity. However, he will offer it only if Jaylor can prove the following claim. Suppose Y is a random variable which only takes on nonnegative integer values. Prove

$$\mathbb{E}[Y] = \sum_{k=0}^{\infty} \Pr[Y > k].$$