

CIS 160 — Mathematical Foundations of Computer Science

Homework Assignment 10T

Assigned: November 2, 2021

Due: 8:30 AM ET, November 4, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, November 4, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:

- 5 points will be deducted from your homework if you do not use the provided L^AT_EX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in L^AT_EX.

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. If multiple solutions are given, only the first one will be graded. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may organize into collaboration teams of up to 3 current students. For each homework assignment, you can only be in one team and must list all team members on your homework submission using the provided L^AT_EX template, whether or not you specifically spoke with them. You may have different teams for different assignments. Collaboration must be strictly limited to discussion, and solutions must be written separately. For the complete collaboration policy, please consult the announcement on Piazza. Violations may seriously affect your grade in the course.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

1. [6 pts] Waterbenders are great with computer science because they love streams

Waterbender and computer science extraordinaire Winnie is hosting a full moon party for the Water Tribe and has designed a game. The game consists of one player rolling a fair six-sided die twice; the player then wins s pieces of blubbered seal jerky where s is the sum of the two rolls. For example, if the player rolls a 2 and then a 3, they will win 5 pieces of seal jerky. Note that each of the 36 possible outcomes of the two rolls is equally likely and has probability $1/36$. Winnie comes up with several conjectures about the game and has asked you answer them. Please justify your answer.

- (a) Let A_i be the event that the first roll is i and let B_j be the event that the second roll is j . Are A_i and B_j independent?
- (b) Let A be the event that the first roll is a 1 and B be the event that the sum of the two rolls is a 7. Are A and B independent?
- (c) Let A be the event that the maximum of the two rolls is 3 and B be the event that the minimum of the two rolls is 3. Are A and B independent?

2. [12 pts] Cat(s)ara's Colorful Cabbages

After a long day of waterbending, Kat(s)ara is craving her favorite food: cabbages. She's in luck, as it just so happens that the world-renowned cabbage merchant is in Towne. He has two boxes of cabbages, regular cabbages and special cabbages and sells his cabbages one by one. Moreover, given the merchant's expertise in cabbages, he has learned to grow cabbages of different colors, and so each box has cabbages of 6 different colors: **Red, White, Yellow, Blue, Orange, and Violet**.

Assume that the cabbage merchant has an equal number of each color of cabbage in each bag, such that he is equally likely to pick a cabbage of each type (**regular** vs **special**) and color pair, (i.e. it is equally probable that the merchant will pull out a **special, red** cabbage as a **regular, violet** cabbage). Further assume that each time the merchant sells a cabbage, he immediately replenishes the bag he took it from with one of the exact same color and type, such that the combination of type and color for each cabbage is independent of the previous cabbage sold.

Kat(s)ara decides to purchase two cabbages.

- (a) Find the probability that both cabbages purchased by Kat(s)ara are **special** given that the first cabbage purchased was a **special, white** cabbage.
- (b) Find the probability that both cabbages purchased by Kat(s)ara are **special** given that at least one of them is a **special, white** cabbage.

3. [12 pts] Season 2 Episode 16 doesn't exist what are you talking about

IshAang's bison Appa has been lost in the Fire Nation! Luckily, the winds have floated a map his way to help him find his wonderfully fluffy companion. On the map, there are $2k$ fire nation military stations and k waterways (a waterway bidirectionally connects two distinct stations) where k is a positive integer. Prove that if the map has no isolated stations, then it has exactly k connected components.