

CIS 160 — Mathematical Foundations of Computer Science

Homework Assignment 10H

Assigned: November 4, 2021

Due: 8:30 AM ET, November 9, 2021

This homework is due electronically on Gradescope at 8:30 AM ET, November 9, 2021. To receive full credit all your answers should be carefully justified.

Please make note of the following:

A. Standard Deductions:

- 5 points will be deducted from your homework if you do not use the provided L^AT_EX template.
- 5 points will be deducted from your homework if you do not select pages when submitting to Gradescope.
- No credit will be awarded to assignments that are not typeset in L^AT_EX.

B. Solutions: Please make sure to keep your solutions clear and precise. While no points will be deducted for overly verbose solutions, clarity and brevity are important skills that can be developed through CIS 160. If multiple solutions are given, only the first one will be graded. *Solutions must be given in closed form (as defined on Piazza).*

C. Collaboration: You may organize into collaboration teams of up to 3 current students. For each homework assignment, you can only be in one team and must list all team members on your homework submission using the provided L^AT_EX template, whether or not you specifically spoke with them. You may have different teams for different assignments. Collaboration must be strictly limited to discussion, and solutions must be written separately. For the complete collaboration policy, please consult the announcement on Piazza. Violations may seriously affect your grade in the course.

D. Citations: All solutions must be written in your own words. If you would like to use part of a solution from a problem presented in lecture, recitation, or past homework solutions you may do so with attribution; i.e., provided you add a comment in which you make clear you copied it from these sources.

E. Outside Resources: Any usage of resources outside of the course materials on the course website or Canvas is strictly prohibited. Violations may seriously affect your grade in the course.

F. Late Policy: We will allow you to drop two homework assignments assigned on a Tuesday and two homework assignments due on a Thursday (i.e. two ‘T’ homeworks and two ‘H’ homeworks). Because of this, we will not accept late homework under any circumstances. If you will be missing school for an extended period of time due to severe illness, please notify the professor.

0. [10 pts] Mid-Semester Feedback Form

Please complete the mid-semester feedback form under Assignments on Canvas. We greatly value your feedback and concerns and want to improve CIS 160 in any way we can. Please note that all of your responses are completely anonymous, so please give us your honest feedback!

1. [10 pts] Who is the funniest grAangkid?

Airbender Ryaang's granddaughter Jaynora has decided to use her airbending skills to fly to a far-off mythical place she has heard of called San Francisco. However, she decides she needs lots of snacks along the way. She goes to visit her brother Mukeelo to ask for some vegetarian sushi.

Mukeelo proposes a game: if Jaynora decides to decline playing the game, Mukeelo will give Jaynora 10 sushi rolls. Otherwise, Jaynora will play the following game:

The game uses two objects: a fair coin and a fair 100-sided die. One side of the coin is labeled with a 1 and the other side is labeled with a 2. Each side of the 100-sided dice roll is labeled with a distinct integer in the interval from 1 to 100, inclusive.

The game consists of two rounds: a coin flip and a 100-sided dice roll. During the first round, Jaynora can choose to either flip the coin or to roll the dice. If whichever she chooses results in a 1, she wins 1 sushi roll and moves on to the next round; if she gets any other number, she must give Mukeelo 10 sushi rolls and forfeit the game, failing to move on to the next round.

If she makes it to the second round, she must use the other object. For instance, if she rolled the dice in the first round, then she must flip the coin in the second round, and vice versa. In the second round, if she gets a 1, she wins 4560 sushi rolls. If she gets any other number, she loses and must give Mukeelo 10 sushi rolls.

After hearing the rules, Jaynora has three options. She can:

- (a) Decide not to play the game.
- (b) Play the game and choose to flip a coin for the first round.
- (c) Play the game and choose to roll the dice for the first round.

Find the expected value of each of these options. Which option has the highest expected value in sushi rolls?

2. [12 pts] Objectively speaking, entering Earth Rumble VI against Toph does not seem like a good idea.

While the TAs are in the Earth Kingdom, they hear about the Earth Rumble VI, an underground earth bending tournament. They decide to send only the strongest present to participate in

hopes of standing a chance against Toph, the reigning champion from last year. To determine who will participate, each of the $n \geq 2$ TAs will fight every other TA exactly once such that each fight has a winner and a loser. They define an “elite TA” w to be a TA such that, for every other TA t , at least one of the following conditions holds:

- (1) w beat t when they fought.
- (2) w beat some other TA t' , and t' beat t when they fought.

Show that there will always be someone who satisfies the above conditions, regardless of the number of TAs and the results of the individual matches.

3. [14 pts] **Bolinda’s Brother Bamboozling Betting Bugaboo**

Bolinda, wanting to prove he’s the smartest in the family, challenges his older brother Muko, to a game of the wits. The game consists of creating a 3×3 grid of fair coins. Each coin is tossed independently and comes up either heads (H) or tails (T). Bolinda needs to bet on the probability of having at least one 2×2 square of heads (H’s) in the grid to beat his brother. Help him find the probability of this event.

4. [10 pts] **Tea? Treason?? TrEAson???**

Uncle Iyer-oh is loving his time in Ba Sing Se because there is no war in Ba Sing Se, and he has decided to spend some time exploring all the tea shops the city has to offer (He has to check out the competition). Ba Sing Se has $2n$ tea shops where $n \geq 1$. There are bidirectional monorails connecting pairs of tea shops, and any pair of tea shops can have at most one monorail between them. Uncle Iyer-oh noticed that no three tea shops have all possible monorails between them. Specifically, for any three tea shops t_1, t_2, t_3 , at least one pair of these tea shops does not have a monorail between them. Prove that Ba Sing Se has at most n^2 monorails.

5. [12 pts] **Ky Lee can walk in both directions along a tightrope? I can’t even go one way!**

Ky Lee, embracing her fleeting youth, walking around from circus to circus having fun. She wants to visit $n \geq 3$ circuses, and each circus can be reached from every other circus by a sequence of tightrope segments, and there is at most one tightrope segment connecting any two circuses. Ky Lee can walk in both directions along any tightrope segment and each tightrope segment must connect exactly two distinct circuses.

Ky Lee is worried about wasting too much energy traveling around rather than having fun doing acrobatics at the circus (and also she doesn’t want to upset Azula). She especially does not want to waste energy by getting lost and walking in circles, visiting the same circuses. Help

reassure her by proving that there exists exactly one cycle of tightrope segments if and only if there are a total of n tightrope segments.

6. [12 pts] There's an old story about a secret pass. Right. Through. The mountains.

Trying to reach Omashu, Yuyaang and Zhatara eventually reach the secret tunnel! At the secret tunnel, Zhatara challenges Yuyaang to a special game.

Zhatara has 50 distinct stones set up around the tunnel. Yuyaang is given one coin. In each round, Yuyaang will hide the coin below one of the 50 stones and bet some amount of money. Zhatara will choose a stone uniformly at random, and if Yuyaang's coin is below that stone, Zhatara will pay Yuyaang so that he makes a net profit of 47 times his initial bet. If Yuyaang's coin is not below the stone Zhatara chose, Yuyang will lose his initial bet.

Since Yuyaang's favorite number is 20, Yuyaang decides that for each round, he is going to place his coin under the 20th stone and bet \$1.

Zhatara then adds a twist to this game. Zhatara bets Yuyaang \$25 that after 48 rounds of the game, Yuyaang will have lost more money than he gained. In other words, Yuyaang will pay Zhatara \$25 if Yuyaang is behind after 48 rounds; otherwise, Zhatara will pay Yuyaang \$25.

- (a) Calculate Yuyaang's expected gain for the 48 rounds only, without Zhatara's \$25 bet. (If he is expected to lose money, his expected gain will be a negative number).
- (b) Calculate the probability that Yuyaang is behind at the end of 48 rounds.
- (c) Calculate Yuyaang's expected gain in his bet against Zhatara (not including the money earned/lost during the game).
- (d) Calculate Yuyaang's overall expected gain from the game and the bet with Zhatara. Does Zhatara's bet dissuade Yuyaang from playing?