delta

Ordered Types for Stream Processing

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January 10, 2024
TFP

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Stream Processing

What is a Stream?

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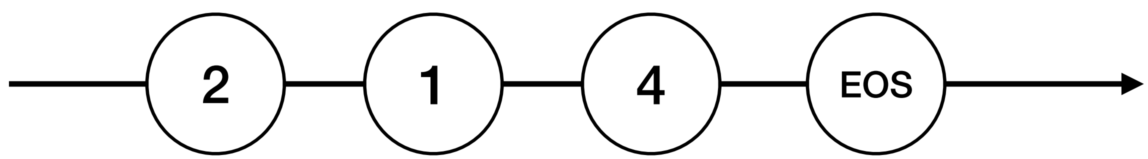
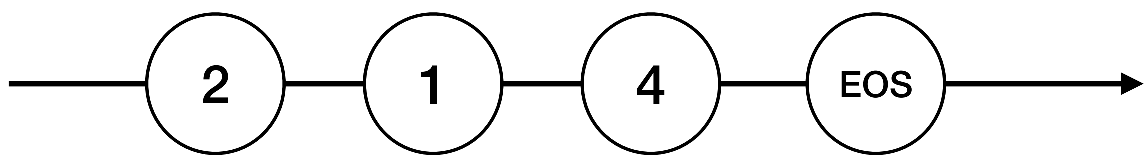
An ordered sequence



What is a Stream?

An ordered sequence

arriving incrementally



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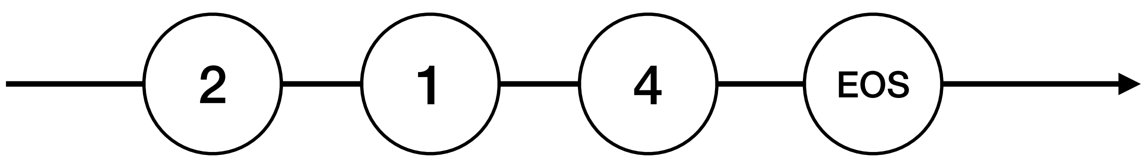
and possibly unbounded in size.



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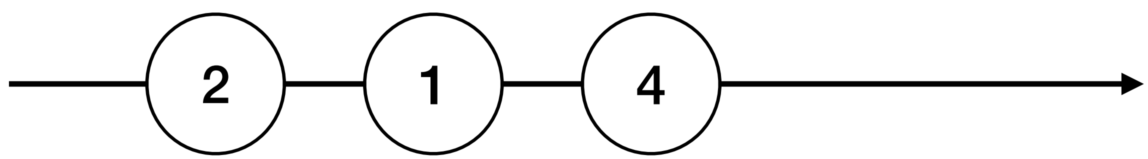

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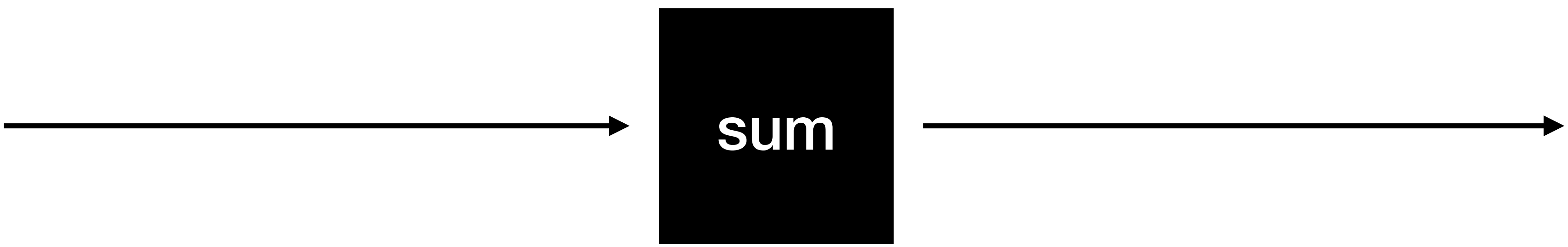

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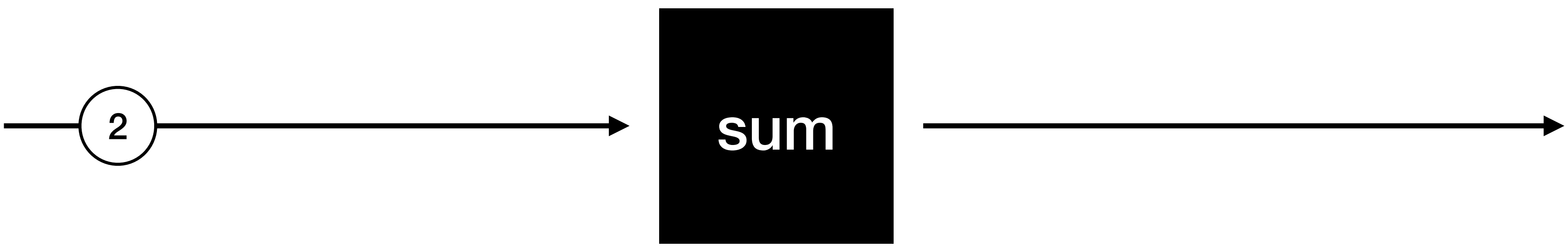

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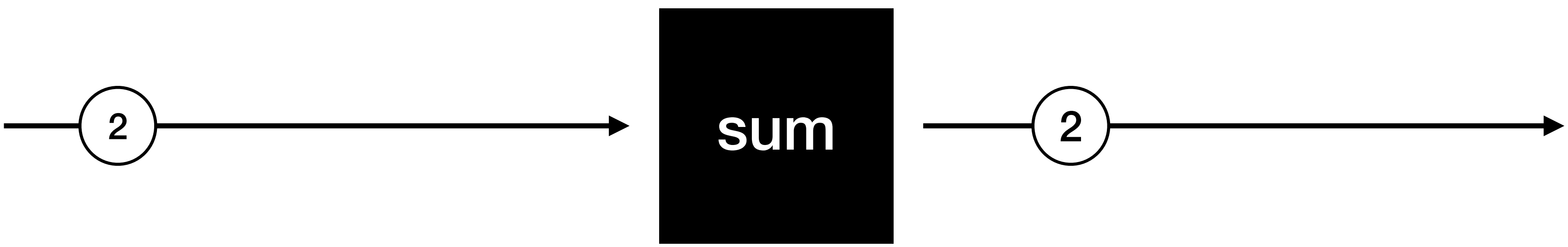

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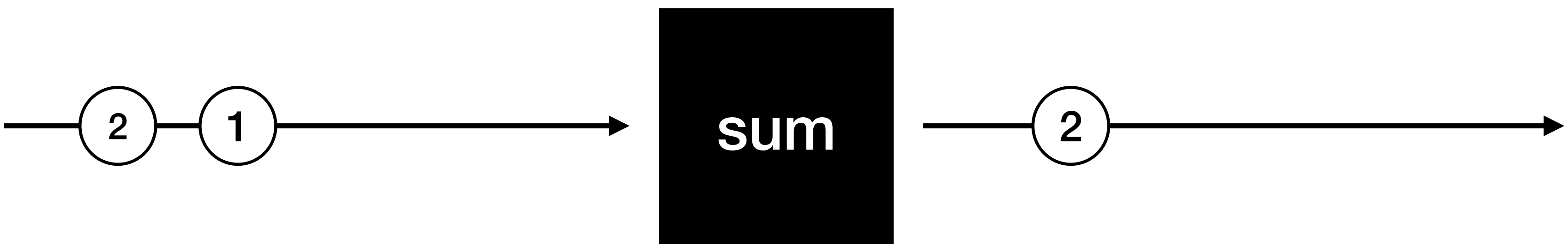

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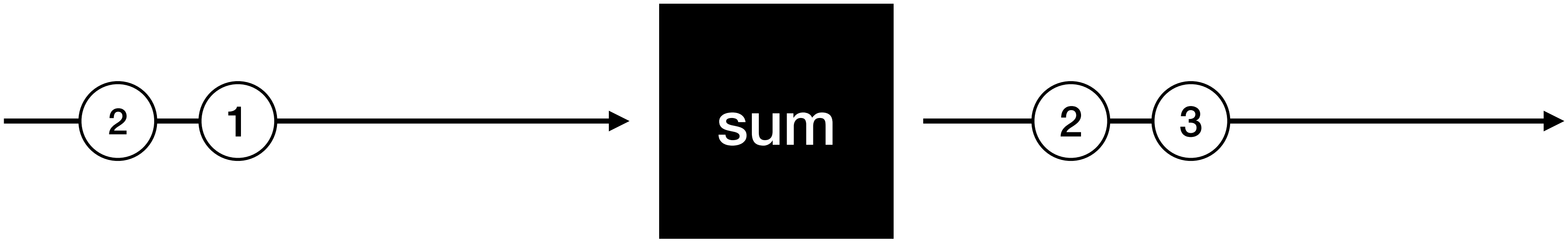

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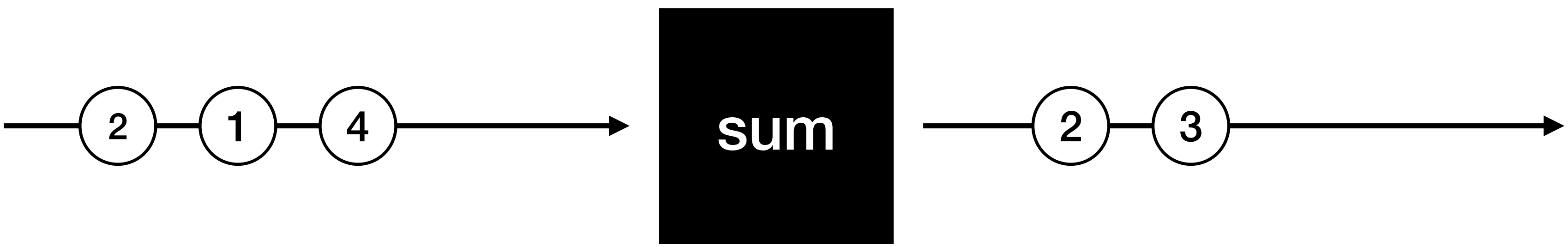

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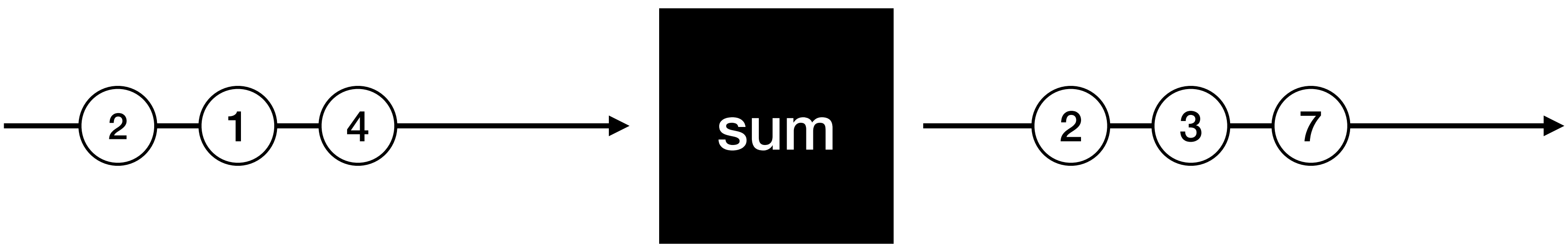

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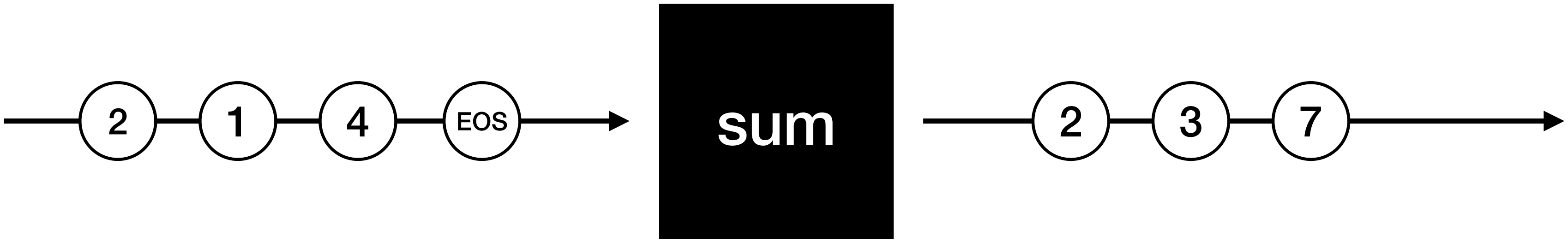

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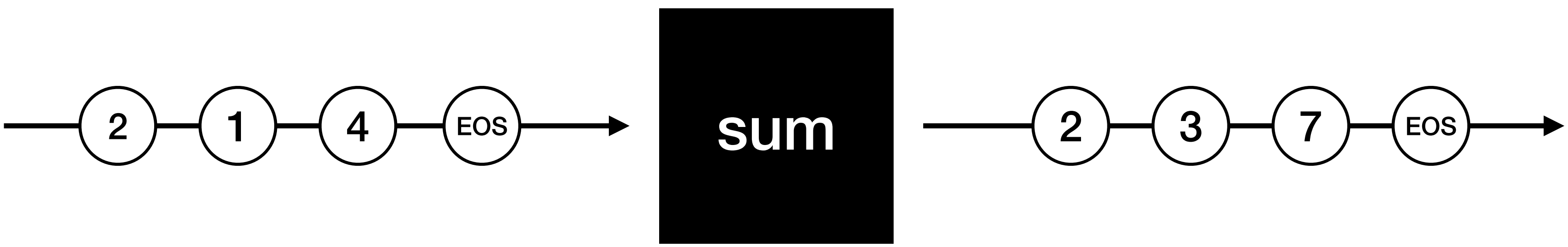

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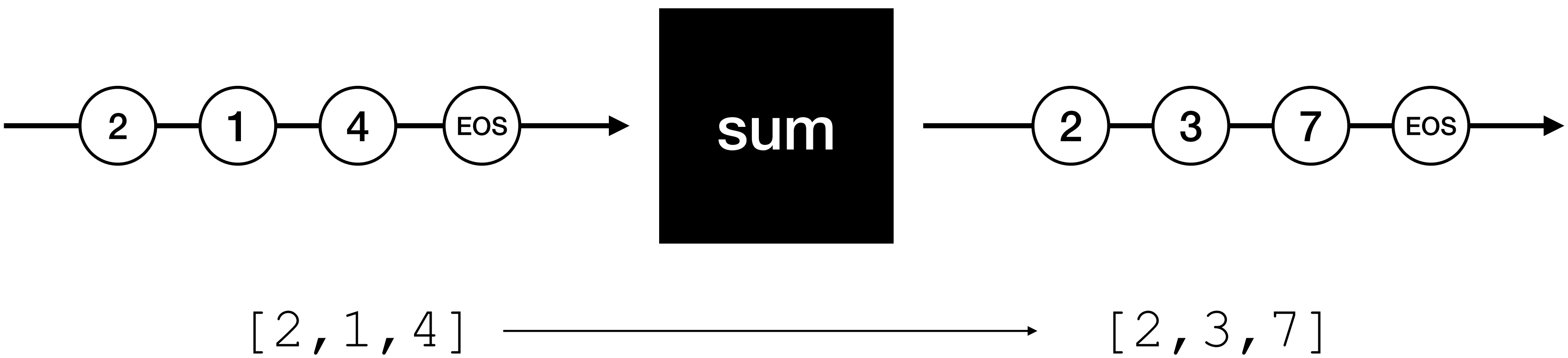

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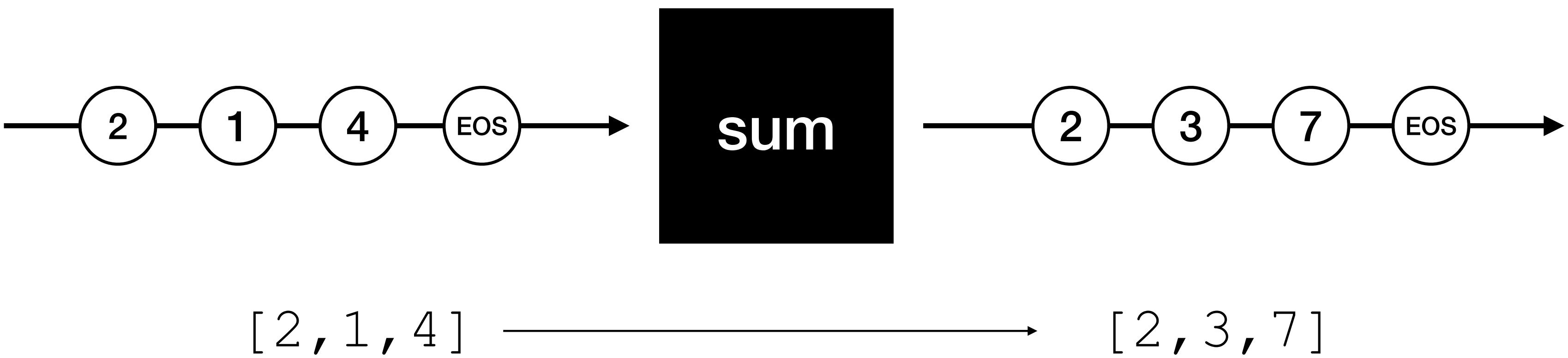
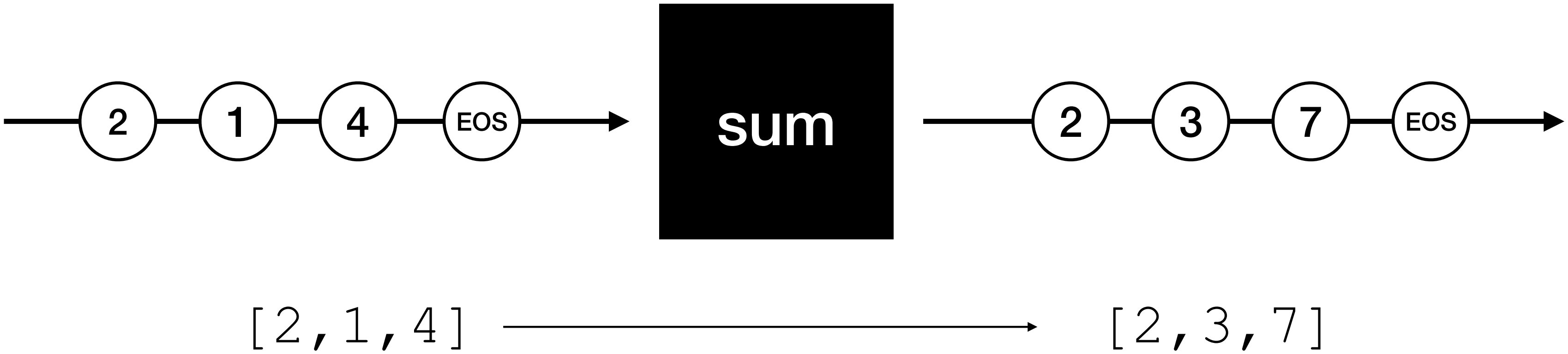


Incremental List Processing

What is Stream Processing?



Incremental List Processing

(with bounded memory)

Programming Models for Streams

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Manual State Machine

```
's -> 'a -> 's * ('b list)
```

❌ Low-Level

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Functional Reactive Programming

```
time -> 'a
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✅ High-Level ❌ Not Resource-Aware

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✅ High-Level ❌ Not Resource-Aware

Streaming eDSL

```
map : ('a -> 'b) -> 'a stream -> 'b stream
filter : ('a -> 'b) -> 'a stream -> 'b stream
window : ('a list -> 'b) -> int -> 'a stream -> 'b stream
fold : ('a -> 'b -> 'b) -> 'b -> 'a stream -> 'b stream
```

✅ Resource Aware

✅ High-Level

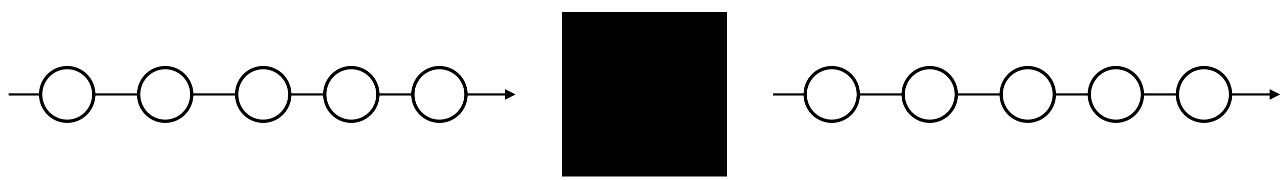
❌ Not Expressive

What if I told you…

You can **write** a natural function on lists…

```
fun partialSums(acc : int, xs : int list) : int list =
  case xs of
    nil => acc
  | y::ys => let acc’ = y + acc in
                 acc’::partialSums(acc’,ys)
```

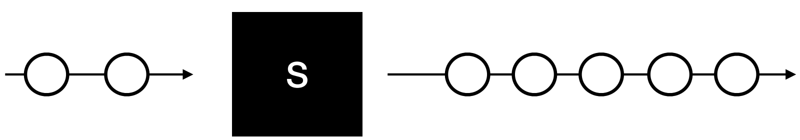
… and **run** it as a streaming program?



delta

Familiar functional syntax…

```
fun foo(xs : int list) =
   case xs of
     nil => …
     y :: ys => … foo(ys) …
```



… with streaming semantics!

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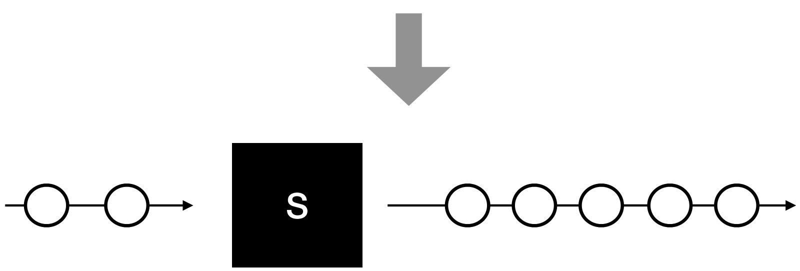
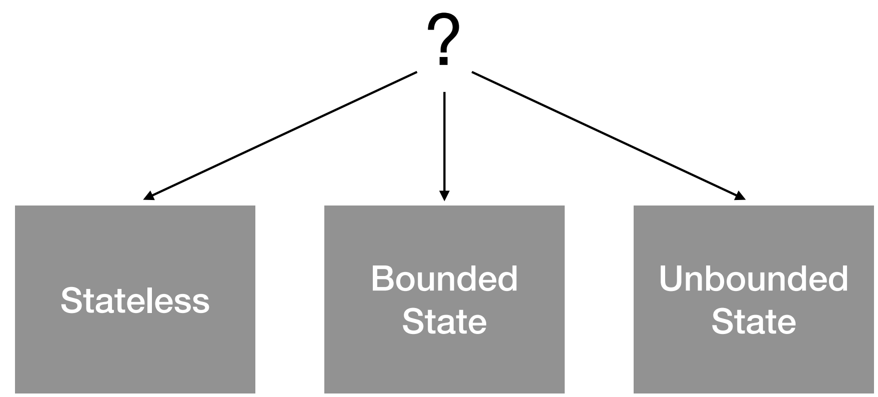


… with streaming semantics!

✅ Expressive ✅ High-Level

And static prevention of space leaks

```
fun foo(xs : int list) = …
```



✅ Resource-Aware

How?

How?

With an *Ordered* Substructural Type System!

A *Non*-Example, for Inspiration

```
fun reverse(xs : s*) : s* =
 case xs of
   nil => nil
 | y :: ys => snoc(reverse(ys);y)
```

"Stream" is written with a star in delta, like regular expressions

A *Non*-Example, for Inspiration

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`y` arrives before `ys`…

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"Stream" is written with a star in delta, like regular expressions

`y` arrives before `ys`…

… but is sent after

We have to save the *entire stream* in memory!

Core Idea:

If all variables are used *in order of arrival of the corresponding data*, then the program is streamable with no auxiliary memory

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If all variables are used *in order of arrival of the corresponding data*, then the program is streamable with no auxiliary memory

delta is **stateless by default**

(programmers can selectively introduce state when needed)

Typing Judgment

$$\underbrace{\Gamma}_{\text{Input Types}} \vdash \underbrace{e}_{\text{Stream Program}} : \underbrace{S}_{\text{Output Type}}$$

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$$\Gamma = \underbrace{(x_0 : S_0)}_{\text{First variable to arrive}}; (x_1 : S_1); \ldots ; \underbrace{(x_n : S_n)}_{\text{Last variable to arrive}}$$

(Questions?)

Variables

$$\frac{}{\Gamma; (x : S)\,; \Gamma' \vdash x : S}$$

Variables

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$$\boxed{x}$$

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$$\xrightarrow{\Gamma} \qquad \qquad \boxed{x}$$

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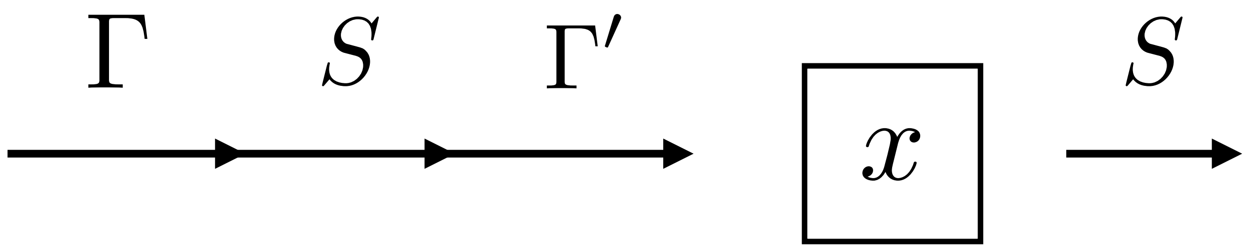

Nil and Cons

$$\frac{}{\Gamma \vdash \mathtt{nil} : S^{\star}}$$

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$\boxed{\texttt{nil}}$

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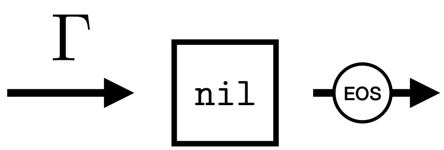

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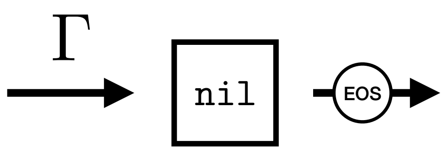
$$\frac{}{\Gamma \vdash \texttt{nil} : S^\star}$$

$$\frac{\Gamma \vdash e : S \qquad \Delta \vdash e' : S^\star}{\Gamma; \Delta \vdash e :: e' : S^\star}$$



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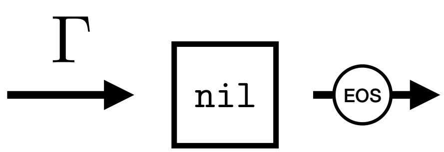

Early inputs used for **first output**

Later inputs used for **rest of outputs**

$$\frac{\Gamma \vdash e : S \qquad \Delta \vdash e' : S^\star}{\boxed{\Gamma ; \Delta} \vdash e :: e' : S^\star}$$

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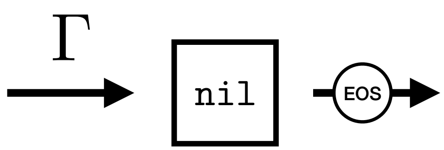

$$\boxed{e :: e'}$$

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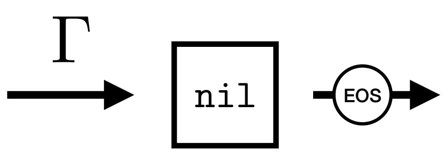

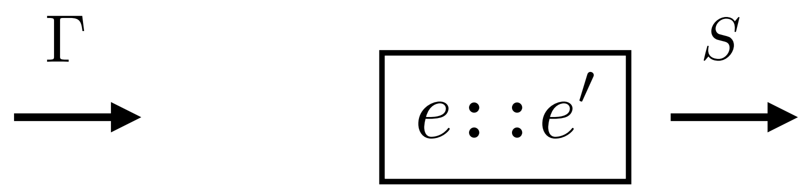

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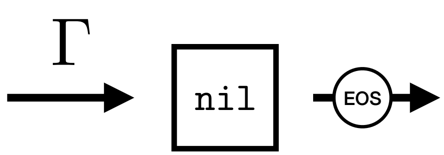

Star-Case

$$\frac{\Gamma; \Gamma' \vdash e : T \qquad \Gamma; (y : S); (ys : S^\star); \Gamma' \vdash e' : T}{\Gamma; (xs : S^\star); \Gamma' \vdash \texttt{case}(xs, e, y.ys.e') : T}$$

Filter

Looks like **list** filter, runs like **stream** filter!

```
fun filter[s](p : s -> Bool)(xs : s*) : s* =
  case xs of
    nil => nil
  | y::ys => let zs = filter(p)(ys) in
             if p(y) then y::zs else zs
```

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```

We can recover all the standard list combinators as stream combinators in this style!

But Wait, There's More!

Ordered Structure Yields Rich Stream Types

$$S \cdot T$$

Concatenation Type

Ordered Structure Yields Rich Stream Types

$$S + T$$

Sum Type

$$S \cdot T$$

Concatenation Type

Ordered Structure Yields Rich Stream Types

$$S + T$$

Sum Type

$$S \cdot T$$

Concatenation Type

$$\texttt{Int}$$

Singleton Type

Ordered Structure Yields Rich Stream Types

$$S + T$$

Sum Type

$$S \cdot T$$

Concatenation Type

$$\texttt{Int}$$

Singleton Type

$$\mathcal{E}$$

Empty Stream Type

Quiz

What streams do these types describe?

$$\texttt{Int} + \texttt{Int} \cdot \texttt{Int} \qquad \texttt{Int} \cdot (\texttt{Int} + \varepsilon)$$

Technical footnote:

Stream Types **vs.** **Separation Logic**

$$S \cdot T$$

Separation in time

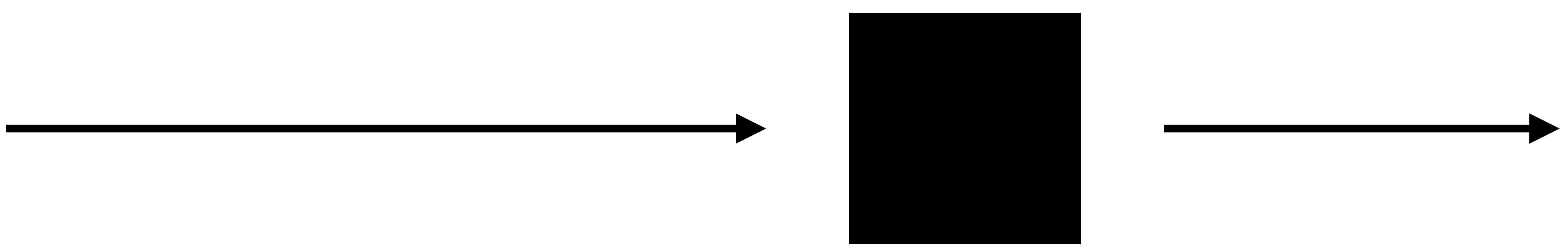
$$S * T$$

Separation in (heap) space

**Rich types
let us build stream programs
*compositionally***

Example

Compute the average of each run above 3



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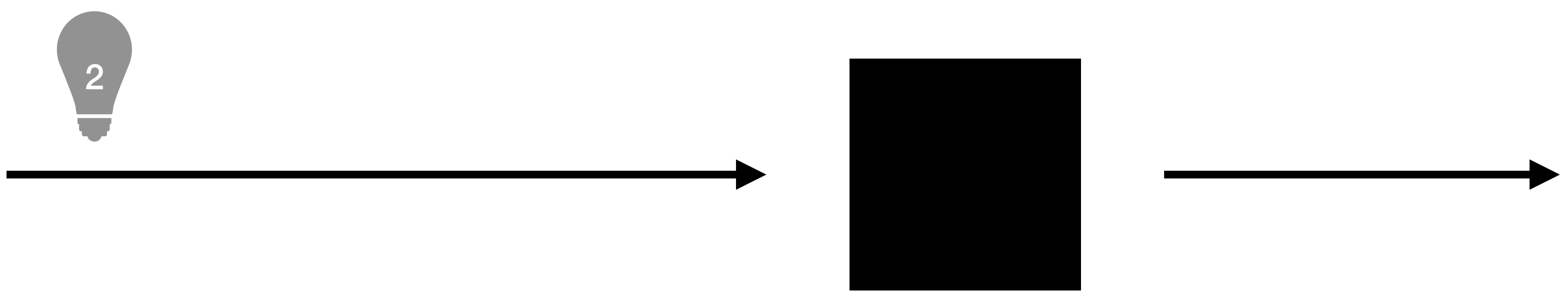

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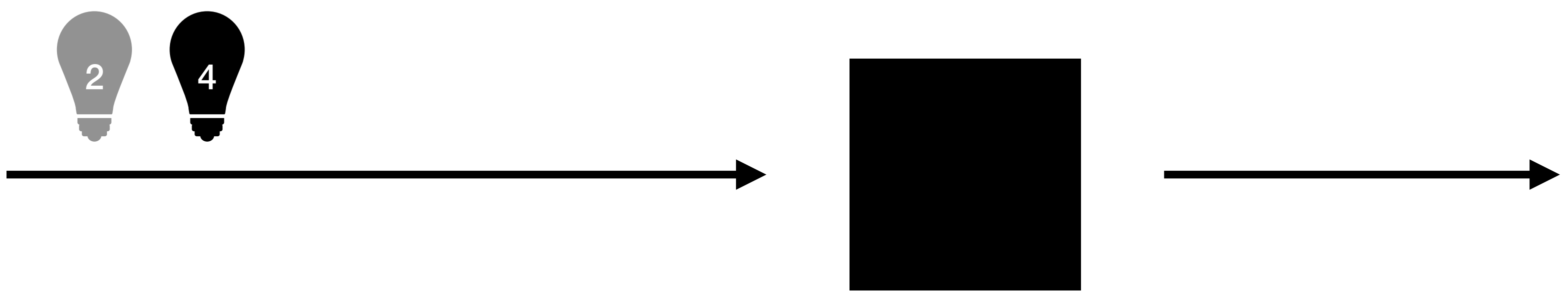

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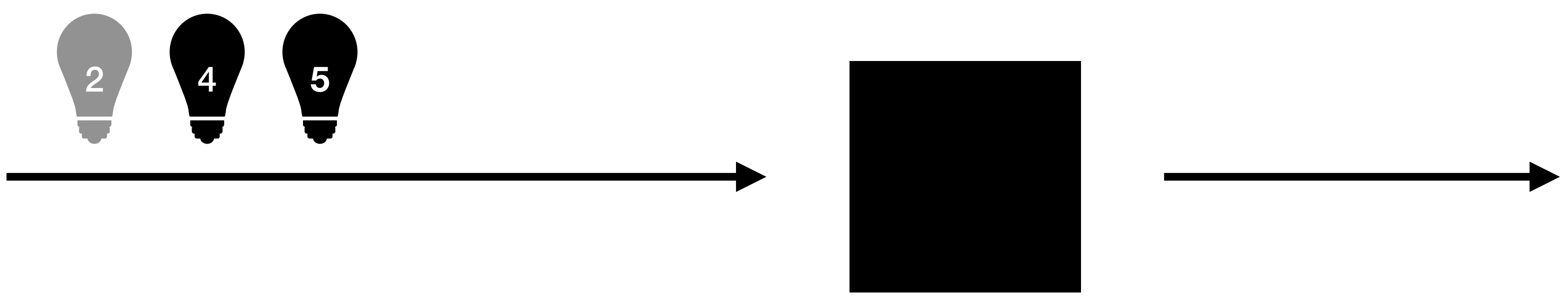

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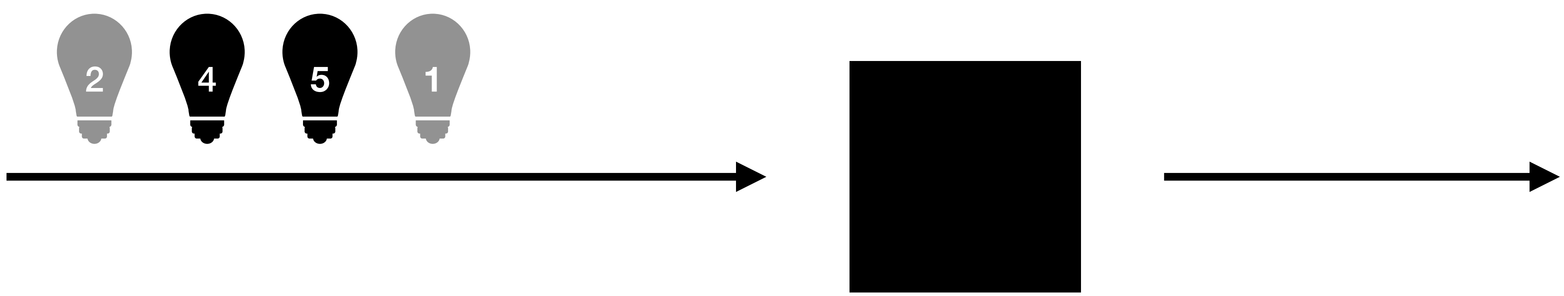

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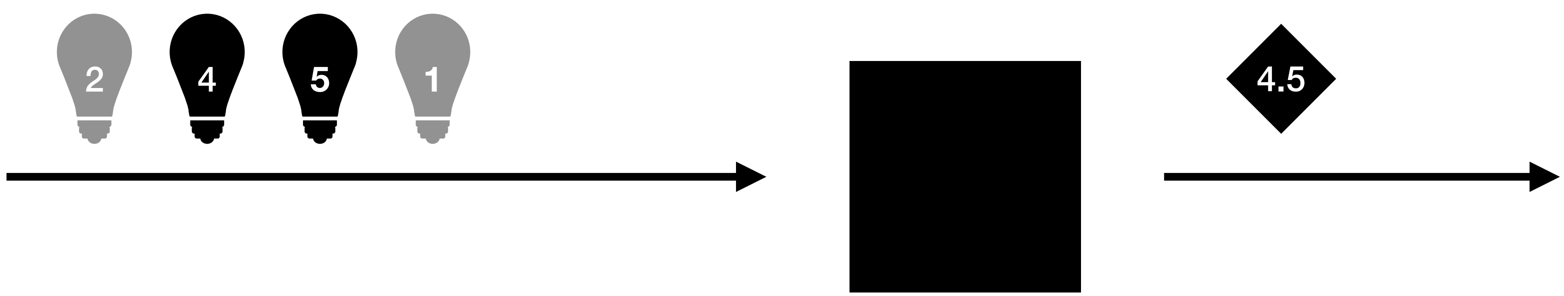

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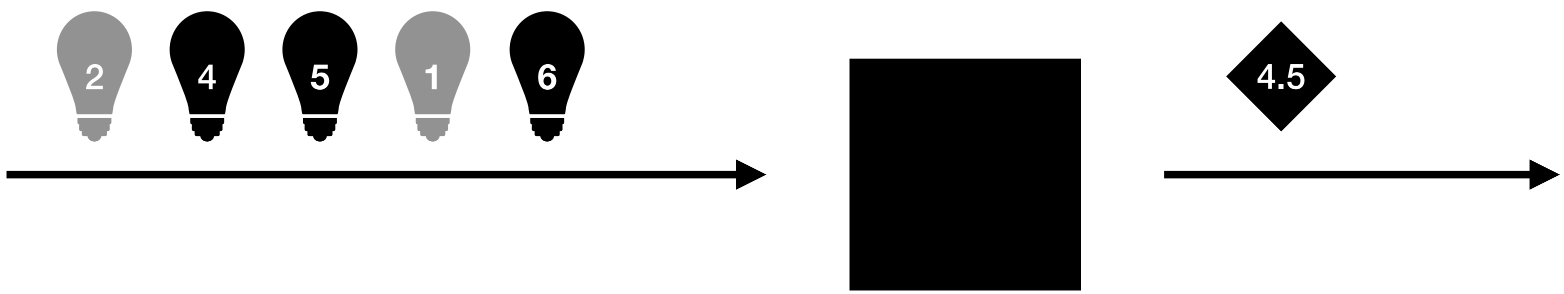

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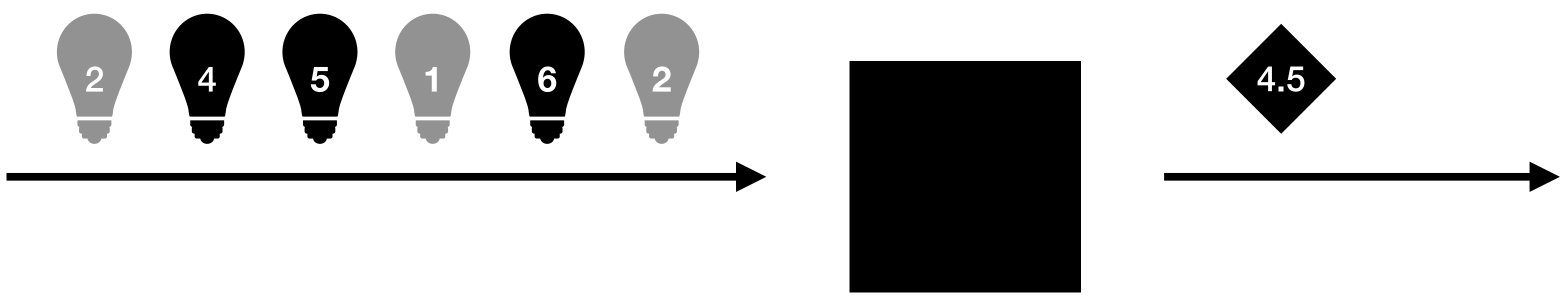

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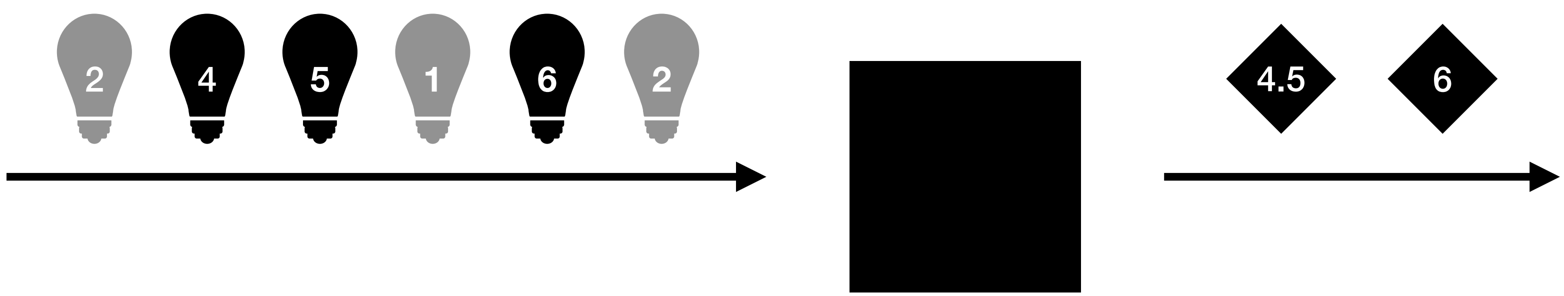

In Flink:

“Compute averages of runs above 3”

In Flink:

"Compute averages of runs above 3"

```
xs.flatMapWithState((x : Int, st : Option[Int,Int]) =>
  st match {
    case None => if x > 3 then ([],Some(1,x)) else ([],None)
    case Some(num, tot) =>
      if x > 3 then
        ([],Some(num + 1, tot + x))
      else
        ([(x + tot) / (num + 1)], None)
  }
)
```

In Flink:

“Comp … bove 3”

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xs.flatMapWithState(
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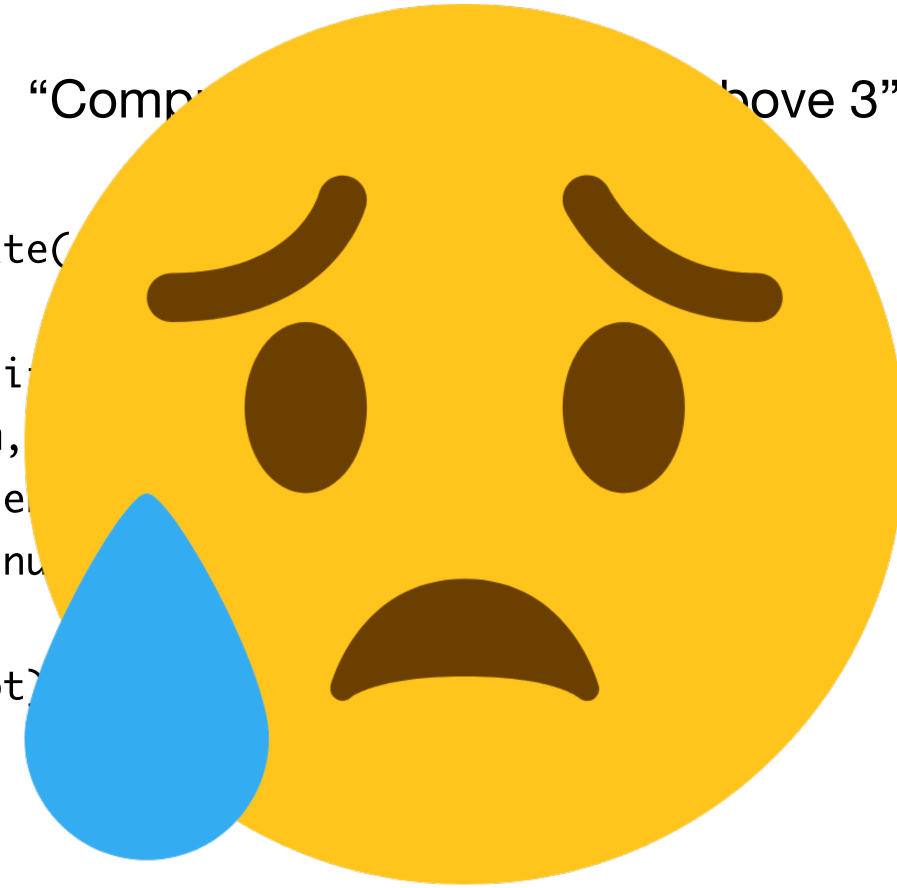

In delta:

delta gives you **types** to help think about the problem!

$$\mathtt{Int} \cdot \mathtt{Int}^\star$$

“Nonempty run of Ints”

$$(\mathtt{Int} \cdot \mathtt{Int}^\star)^\star$$

“Stream of nonempty runs of Ints”

Typeful Programming in delta

$$\texttt{Int} \cdot \texttt{Int}^{\star} \xrightarrow{\texttt{avgRun}} \texttt{Int}$$

$$\begin{tikzcd} \texttt{Int}^{\star} \arrow[r, "\texttt{parseRuns}"] \arrow[rr, bend right, dashed, "\texttt{avgAbove3}"'] & (\texttt{Int} \cdot \texttt{Int}^{\star})^{\star} \arrow[r, "\texttt{map(avgRun)}"] & \texttt{Int}^{\star} \end{tikzcd}$$

Typeful Programming in delta

```
fun average1Run(w : Int . Int*) : Int =
  let (x;xs) = w in
  let (k,n) = (sum(xs), length(xs)) in
  (k + x) / (n + 1)

fun avgAbove3(XS : Int*) : Int* =
  map(average1Run)(parseRuns(geq3)(xs))
```

Typeful Programming in delta

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```

Exact same semantics as the fold in Flink!

But Wait, There’s *Even* More!

What about multiple parallel inputs?

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What about multiple parallel inputs?

$$S_1 \| S_2 \| S_3 \longrightarrow \blacksquare \longrightarrow T_1 \| T_2$$

Multiple inputs are products!

“Parallel Substreams” Type

$$S \| T$$

"A stream of type S interleaved with a stream of type T”

(with elements tagged to indicate their source)

Careful…

Parallel inputs risk **nondeterminism!**

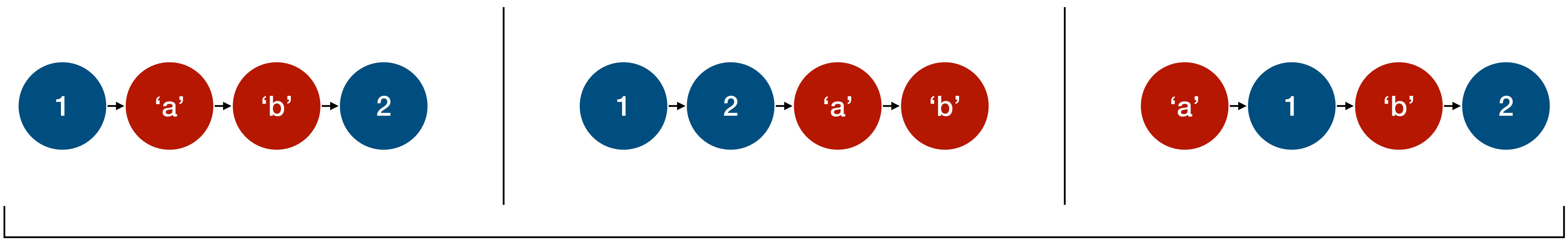
Problem: Multiple Equivalent Interleavings

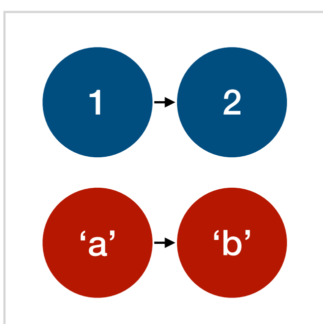
$$\texttt{Int}^\star \,\|\, \texttt{Char}^\star$$

1 → ‘a’ → ‘b’ → 2 | 1 → 2 → ‘a’ → ‘b’ | ‘a’ → 1 → ‘b’ → 2

Problem: Multiple Equivalent Interleavings

$$\mathtt{Int}^{\star} \parallel \mathtt{Char}^{\star}$$





Streams with parallel are actually *partially ordered*

We <u>don't</u> want to be able to write this:

```
fun imposeOrder(x : Int || Char) : Int =
  <… if the Int arrives first,
        send it along and drop the Char;
     if the Char arrives first,
        send 42 and drop the Int…>
```

Core Idea 2:

If variables corresponding to unordered parts of the data are *never used in an ordered way,* the program is deterministic

Core Idea 2:

If variables corresponding to unordered parts of the data are *never used in an ordered way,* the program is deterministic

delta is **deterministic by default**

(programmers can selectively introduce nondeterminism)

Parallel Right Rule

$$\frac{\Gamma \vdash e : S \qquad \Gamma \vdash e' : T}{\Gamma \vdash (e, e') : S \| T}$$

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$$\boxed{(e, e')}$$

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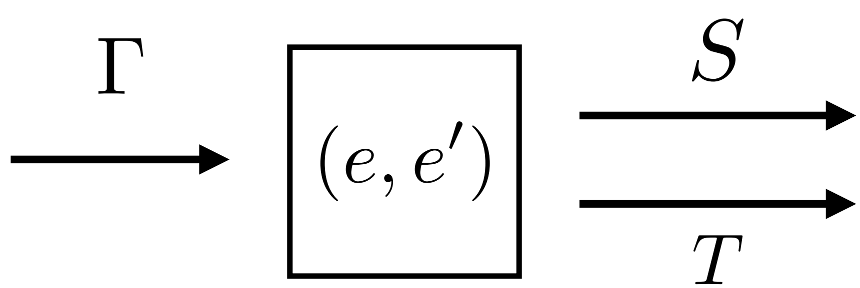

Parallel Left Rule, Take 1

$$\frac{\Gamma; \texttt{???}; \Gamma' \vdash e : R}{\Gamma; z : S \| T; \Gamma' \vdash \texttt{let}\,(x, y)\; = \; z\,\texttt{in}\,e : R}$$

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$(x : S); (y : T)$

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$(x : S)\cancel{,}(y : T)$ $\qquad\qquad$ $(y : T); (x : S)$

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$(x : S)\cancel{,}(y : T)$

$(y : T)\cancel{,}(x : S)$

Parallel Left Rule, Take 1

$$\frac{\Gamma; \mathtt{???}; \Gamma' \vdash e : R}{\Gamma; z : S \| T; \Gamma' \vdash \mathtt{let}\,(x, y) \ = \ z\ \mathtt{in}\ e : R}$$

$$(x : S) \cancel{,} (y : T) \qquad (x : S), (y : T) \qquad (y : T) \cancel{,} (x : S)$$

“Bunched” Contexts

$$\Gamma ::= \cdot \mid x : S \mid \Gamma, \Gamma \mid \Gamma; \Gamma$$

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Γ, Δ	$\Gamma; \Delta$
Unordered	Ordered

“Bunched” Contexts

$$\Gamma ::= \cdot \mid x : S \mid \Gamma, \Gamma \mid \Gamma; \Gamma$$

Γ, Δ	$\Gamma; \Delta$
Unordered	Ordered
Corresponds to parallel	Corresponds to concat

Unordered data can't be used in an ordered way

$$\frac{\vdots}{(x : S)\,,(y : T) \vdash (x; y) : S \cdot T}$$

Formally: Cat-R rule
requires semicolon context

Unordered data can't be used in an ordered way

$$\frac{\vdots}{(x : S)\,,\,(y : T) \not\vdash (x; y) : S \cdot T}$$

Formally: Cat-R rule
requires semicolon context

A Stream Partitioner

Looks like **list** partition, runs like **stream** partition!

```
fun partition[s](p : s->Bool)(xs : s*) : s*||s* =
  case xs of
    nil => nil
  | y::ys => let (zs,ws) = partition(p)(ys) in
                 if p(y) then (y::zs,ws) else (zs,y::ws)
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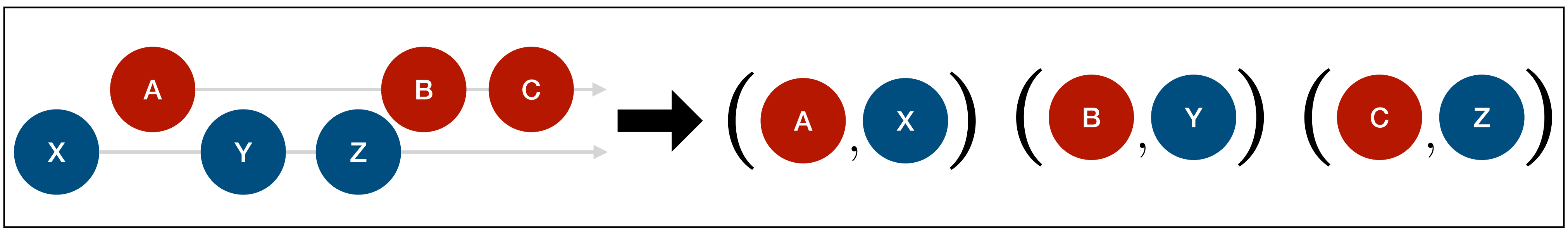
```

🎉 Again, we can recover many standard parallelism combinators as stream programs in this style! 🎉

Deterministic Merging of Parallel Streams

```
fun merge[s](xs : s*, ys : s*) : (s || s)* =
  case xs of
    nil => nil
  | x'::xs' =>
      case ys of
        nil => nil
      | y'::ys' => (x',y') :: merge(xs',ys')
```



Execution model

Today

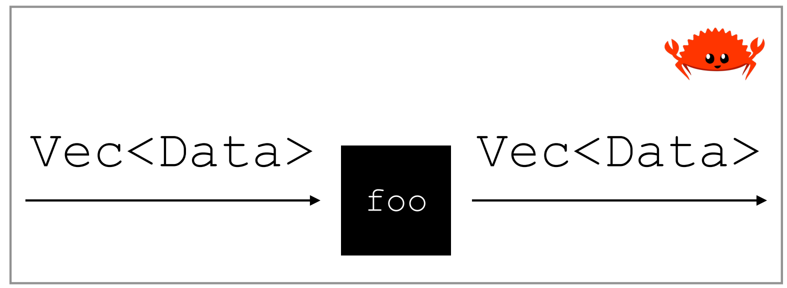
Single-node
Haskell interpreter

Soon

Compiler to Rust-based
runtime above Hydro
dataflow engine

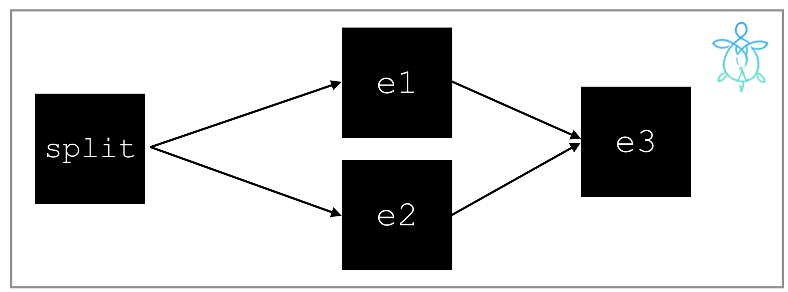
Single-Node Compiler

```
fun foo(x : s) : t = …
```



Multi-Node Compiler

```
fun foo(x : s) : t =
   let (y,z) = (e1,e2)
   in e3
```



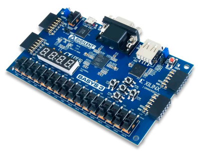
Wrapping up…

Future Directions

Compile Targets

Hydro

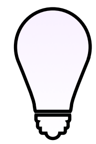
Distributed Systems



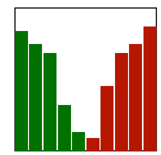
Hardware

Streaming DBs

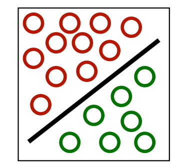
Case Studies



IoT and Edge



Financial Data



ML Streaming

Theory

$[\![\cdot]\!]$

Denotational Semantics

`'a signal`

FRP Connections

$\approx$

Rewriting & Optimization

<Your Application Here>

Thank you!!

jwc@seas.upenn.edu
bcpierce@seas.upenn.edu

https://www.seas.upenn.edu/~jwc/assets/stream-types.pdf

https://github.com/alpha-convert/delta

Questions?