Learning Algorithms From Game Theory II: Boosting

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Overview

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We’ll focus on the general and empirically successful paradigm of *boosting*.

Boosting addresses the question of how one can combine classifiers that individually do (just) a little bit better than random guessing, into powerful predictive models.
Setting

Definition
A labeled *datapoint* is a pair \((x, y) \in X \times Y\), where \(X\) is some space of *features* and \(Y\) is some space of *labels*: for example, a common case is \(X = \mathbb{R}^d\), and \(Y = \{0, 1\}\). A dataset \(D \in (X \times Y)^n\) is a collection of \(n\) labeled datapoints.
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Setting

Definition
Given a predictor $f : X \rightarrow Y$, its prediction accuracy on a dataset $D$ is:

$$\text{acc}(f, D) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}[f(x_i) = y_i]$$

The prediction accuracy as defined uniformly weights all of the points in the dataset. But we can also define weighted prediction accuracy relative to any other weighting $w \in \Delta[n]$ of the $n$ points:

$$\text{acc}(f, D, w) = \sum_{i=1}^{n} w_i \mathbb{1}[f(x_i) = y_i]$$

Note that $\text{acc}(f, D)$ is simply the special case of $\text{acc}(f, D, w)$ in which $w_i = 1/n$ for all $i$. 
An Aside

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3. Informally, to do this it suffices to predict the labels in $D$ accurately with “simple” hypotheses.
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1. We’re ignoring an important statistical aspect of machine learning!
2. The goal is not to predict the labels of points in our dataset \( D \) (we already know them!) but to predict well on new points drawn from the same distribution.
3. Informally, to do this it suffices to predict the labels in \( D \) accurately with “simple” hypotheses.
4. The boosting approach in this lecture does this, but we’ll just focus on the algorithmic aspects.
A hypothesis class $H$ is a collection of predictors or hypotheses $h : X \to Y$. A weighted learning algorithm $A$ with range $H$ is a mapping from datasets and weight vectors to hypotheses in $H$. $A : (X \times Y)^n \times [0,1]^n \to H$. 

1. If $Y = \{0,1\}$ then it is uninteresting to find a hypothesis $h$ with $\text{acc}(h, D) \leq 1/2$. 
2. Could have done this by random guessing! 
3. Want accuracy more like 0.99... 
4. But what if you can reliably get accuracy 0.51?
5. An algorithm that can guarantee this is a weak learner.
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Weak Learning

**Definition**
A weighted learning algorithm $A$ is a weak learning algorithm for $D$ if for every distribution $w \in \Delta[n]$, $A(D, w) = h$ such that:

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4. So weights are without loss of generality.
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Weak Learning and Strong Learning are Equivalent

Theorem
For any dataset $D$, if there exists an efficient (polynomial time) weak learning algorithm $A$ for $D$, then there exists an efficient strong learning algorithm $A'$ for $D$. 

Proof Idea: Study the appropriately defined zero sum game. Then compute the equilibrium strategy in that game.
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   2.1 The action space for the minimization player (the "Data Player") is the set of datapoints in the dataset: $A_1 = D$. 

   2.2 The action space for the maximization player (the "Learner") is $A_2 = H$.

3. What is the value of this game?
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2. i.e. there is a fixed distribution \( p^* \) over hypotheses \( h \in H \) such that for every data point \( (x_i, y_i) \in D \), at least 51% of the probability mass under \( p \) is on hypotheses that correctly label \((x_i, y_i)\).
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3. So consider the following “majority vote” classification rule \( f_{p^*} \):

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f_{p^*}(x) = 1 \left[ \sum_{h: h(x) = 1} p^*_h \geq 0.5 \right]
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Proof

Lemma

For the distribution $p^* = \max_{p \in \Delta H} \min_{i \in [n]} \sum_{h \in H} p_h \mathbb{1}[h(x_i) = y_i]$, the hypothesis $f_{p^*}$ satisfies $\text{acc}(f_{p^*}, D) = 1$

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3. If \( y_i = 1 \), we have that
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4. Similarly, if $y_i = 0$, we know that $\sum_{h: h(x) = 1} p_h^* < 0.49$ and hence by definition $f_{p^*}(x_i) = 0$. 

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5. We know how to do that by having the data player play polynomial weights over the data points, and the learner best respond.

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4. In fact, we only need an $\epsilon$-approximate equilibrium for $\epsilon < \sqrt{\epsilon}$.
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Algorithm 1 Boost($D, A$)

Let $T \leftarrow \frac{4 \log n}{\epsilon^2}$ for $\epsilon < 0.01$.

Initialize a copy of polynomial weights to run over $w^t \in \Delta^n$.

for $t = 1$ to $T$ do

Let $h^t = A(D, w^t)$

Let $\ell^t \in [0, 1]^m$ be such that $\ell^t_i = 1[h^t(x_i) = y_i]$.

Pass $\ell^t$ to the PW algorithm.

end for

Let $\hat{\rho} = \frac{1}{T} \sum_{t=1}^{T} e_{h^t}$. (Note that this is concisely representable even though $H$ is large, because $\hat{\rho}$ has support over only the $T$ models $h^t$.)

Return $f_{\hat{\rho}}(x)$. 
Proof

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4. Total running time is $O(\log n(n + R(A)))$, where $R(A)$ is the running time of our weak learning algorithm.
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This can be formalized in various ways, including with a measure called "VC-Dimension".
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Briefly: Statistical Aspects

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4. This can be formalized in various ways, including with a measure called “VC-Dimension”.
Thanks!

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Have a great summer!