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- But maybe there is some richer family of equilibria we can shoot for...
- Analogous to our earlier relaxation from *Pure* to *Mixed* equilibria.
Consider the following two player traffic light game that will be familiar to those of you who can drive:

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But one player never gets any utility...
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There is also a mixed strategy Nash equilibrium:

1. Suppose player 1 plays \((p, 1-p)\).
2. If the equilibrium is to be fully mixed, player 2 must be indifferent between his two actions – i.e.:
   \[0 = p - 100(1 - p) \iff 101p = 100 \iff p = 100/101\]
3. So both players play STOP with probability \(p = 100/101\), and play GO with probability \((1-p) = 1/101\).
4. This is even worse! Now both players get payoff 0 in expectation (rather than just one of them), and risk a horrific negative utility.
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5. We can generalize this...
Correlated Equilibrium

Definition

A correlated equilibrium is a distribution $\mathcal{D}$ over action profiles $A$ such that for every player $i$, every action $\hat{a}_i$, and every action $a_i^*$:

$$E_{a \sim \mathcal{D}}[u_i(a)|a_i = \hat{a}_i] \geq E_{a \sim \mathcal{D}}[u_i(a_i^*, a_{-i})|a_i = \hat{a}_i]$$
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In words:

a distribution over action profiles $a$ such that after a profile $a$ is drawn, playing $a_i$ is a best response for player $i$ conditioned on seeing $a_i$, given that everyone else will play according to $a$.

For example: Conditioned on seeing STOP, you know your opponent will GO, so STOP is a best response. Conditioned on seeing GO, you know your opponent will STOP, so GO is a best response.
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Hierarchies

1. Observe: Nash Equilibria are also Correlated Equilibria — they just correspond to uncorrelated distributions. ($a_i$ contains no information about $a_{-i}$).

2. But Correlated Equilibria are a strictly larger/richer set.

3. We can define still larger sets!

Definition
A coarse correlated equilibrium is a distribution $D$ over action profiles $A$ such that for every player $i$, and every action $a^*_i$:

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4. The difference: the suggestion just has to be a best response on average, not conditioned on having seen it.

5. Whether it is sensible depends on whether you have to commit to following the correlating device up front, or have the option of deviating after seeing the suggestion.
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CCE can occasionally suggest obviously bad actions. CE cannot. Consider:

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The payoff for each player for playing according to this distribution is:

\[
\frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 1 - \frac{1}{3} \cdot 1 = 0
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the payoff a player would get by playing the fixed action $A$ or $B$ while his opponent randomized would be:

$$
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and the payoff for playing $C$ would be strictly less than zero.
the payoff a player would get by playing the fixed action $A$ or $B$ while his opponent randomized would be:

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and the payoff for playing $C$ would be strictly less than zero.

Hence this is a CCE *even though* conditioned on being told to play $C$, it is not a best response. This means that the given distribution is a coarse correlated equilibrium, *but not* a CE.
Hierarchies

Solution Concept Recap

\[ DSE \subset PSNE \subset MSNE \subset CE \subset CCE \]
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1. Starting at MSNE, we have guaranteed existence.
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1. Starting at MSNE, we have guaranteed existence.
2. Want to show: Starting at CE, we have computational tractability.
Characterization in Terms of Regret

Definition
For a strategy modification rule $F_i : A_i \rightarrow A_i$ and an action profile $a \in A$:

$$\text{Regret}_i(a, F_i) = u_i(F_i(a_i), a_{-i}) - u_i(a)$$

i.e. it is how much player $i$ regrets not applying $F_i$ to change his action.

We say that $F_i$ is a constant strategy modification rule if $F_i(a_i) = F_i(a'_i)$ for all $a_i, a'_i \in A_i$. 

We can give an alternative characterization of CCE:

Definition
A distribution $D$ is a coarse correlated equilibrium if for every player $i$ and for every constant strategy modification rule $F_i$:

$$E_{a \sim D}[\text{Regret}_i(a, F_i)] \leq 0$$
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1. An immediate consequence of this definition is that if $a^1, \ldots, a^T$ are a sequence of actions with $\Delta(T)$ regret, then
   \[ \bar{a} = \frac{1}{T} \sum_{t=1}^{T} a^t \]
   forms a $\Delta(T)$-approximate coarse correlated equilibrium.
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2. In particular, if everyone plays an (arbitrary) game with the PW algorithm, after \( T \) steps they will have generated a sequence of plays that corresponds to a 
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2. In particular, if everyone plays an (arbitrary) game with the PW algorithm, after $T$ steps they will have generated a sequence of plays that corresponds to a $\Delta(T) = 2\sqrt{\log k/T}$-approximate CCE.

3. Can we approach computing CE in the same way? First step: characterize CE in terms of regret.
Characterization in Terms of Regret

Definition
A distribution $\mathcal{D}$ is a correlated equilibrium if for all players $i$ and for all strategy modification rules $F_i$:

$$E_{a \sim \mathcal{D}}[\text{Regret}_i(a, F_i)] \leq 0$$

To see this:

1. Note that a strategy modification rule $F_i$ lets player $i$ consider different deviations for each suggested action $a_i$.
2. So if there are no beneficial deviations of this sort, player $i$ must be playing a best response even conditioned on seeing his suggestion.
3. Are there learning algorithms that efficiently converge to correlated equilibrium?
4. Look for learning algorithms with stronger regret guarantees...
Characterization in Terms of Regret

Definition
A distribution $\mathcal{D}$ is a correlated equilibrium if for all players $i$ and for all strategy modification rules $F_i$:

$$E_{a \sim \mathcal{D}}[\text{Regret}_i(a, F_i)] \leq 0$$

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Characterization in Terms of Regret

Definition
A distribution $D$ is a correlated equilibrium if for all players $i$ and for all strategy modification rules $F_i$:

$$E_{a \sim D}[\text{Regret}_i(a, F_i)] \leq 0$$

To see this:

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1. We want an algorithm for learning in the experts setting that can promise...
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2. Given any $k$ experts and an arbitrary sequence of losses $\ell^1, \ldots, \ell^T$, the algorithm chooses a sequence of experts $a_1, \ldots, a_t$ such that:

$$\frac{1}{T} \sum_{t=1}^{T} \ell_{a^t} \leq \frac{1}{T} \sum_{t=1}^{T} \ell_{F(a^t)} + \Delta(T)$$

for all strategy modification rules $F$ and for $\Delta(T) = o(1)$.
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3. “No Swap Regret”
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3. “No Swap Regret”

4. We’ll see how to do this! (Next lecture).
Thanks!

See you next class — stay healthy!