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- But many auctions have a more pecuniary goal. What if we want to maximize revenue?
- What does that mean? What is our benchmark?
- This lecture: a case study for single item auctions.
The VCG mechanism was remarkable: we could always maximize welfare \textit{ex-post}. 

What about for revenue? Not so simple.

Consider a single bidder, single item auction. Offering a fixed price \( p \) is always dominant strategy truthful.

Revenue is \( p \) if \( v_i \geq p \), 0 otherwise.

So ex-post, the revenue-optimal auction sets \( p = v_i \) ... But ex-ante, we don't have enough information.
Reasonable Benchmarks?

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The Average Case

- Suppose we know that bidders have valuations $v_i \sim D$ for some distribution $D$. 

- For a single item, single bidder auction, a fixed price $p$ yields expected revenue: 
  
  $$
  \text{Rev}(p) = p \cdot (1 - F(p)) 
  $$

  Where $F(p) = \Pr[v \sim D \mid v \leq p]$. 

- E.g. if $D$ is uniform on $[0, 1]$, then $F(p) = p$ and:

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  \max_p \text{Rev}(p) = \frac{1}{2} \cdot (1 - \frac{1}{2}) = \frac{1}{4}
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Average Case: Many Bidders

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\mathbb{E}_{v \sim D^n} \left[ \sum_{i=1}^{n} P_i(v) \right]
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- And we know:

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P_i(v) = v_i \cdot X_i(v) - \int_{0}^{v_i} X_i(z, v_{-i}) \, dz
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Myserson Optimal Auctions

- Let's assume monotonicity for now, and use our expression for $P$ to derive the optimal $X$.
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- Plan: Find \( X \) to maximize:

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E_{v \sim D^n} \left[ \sum_{i=1}^{n} P_i(v) \right] = \sum_{i=1}^{n} E_{v_{-i} \sim D^{n-1}} \left[ E_{v_i \sim D} [P_i(v_i, v_{-i})] \right]
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**Notation:**
\( f(p) \) is the pdf of \( D \).
\( F(p) = \Pr(v \sim D, v \leq p) = \int_0^p f(v) \, dv \)
Myserson Optimal Auctions

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Myserson Optimal Auctions

Consider the inner term:

\[ E_{v_i} [P_i(v)] = E_{v_i} \left[ v_i \cdot X_i(v_i, v_{-i}) - \int_0^{v_i} X_i(z, v_{-i}) dz \right] \]
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So: We want to maximize
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E_{v \sim D^n} \left[ \sum_{i=1}^{n} \phi(v_i) \cdot X(v) \right]
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\phi(v_i) = \left( v_i - \frac{1 - F(v_i)}{f(v_i)} \right)
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“Virtual Value”
Myserson Optimal Auctions

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$$E_{\nu \sim D^n} \left[ \sum_{i=1}^{n} \phi(v_i) \cdot X(\nu) \right]$$

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- Our objective looks just like welfare with values replaced by virtual values.
Myserson Optimal Auctions

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- (Pointwise) optimal allocation rule: Give the item to the bidder \( i \) with highest \( \phi(v_i) \) if it’s positive. Otherwise give the item to nobody.
Myserson Optimal Auctions

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- (Pointwise) optimal allocation rule: Give the item to the bidder $i$ with highest $\phi(v_i)$ if it’s positive. Otherwise give the item to nobody.

- This is a monotone allocation rule if $D$ is regular: $\phi(v_i)$ is monotone.
  - e.g. if $D$ is uniform, $\phi(v_i) = v_i - (1 - v_i) = 2v_i - 1$
  - Note that $\phi^{-1}(0)$ recovers the optimal $p = 1/2$ for a single bidder.
What do revenue maximizing auctions look like? (when \( v_i \) drawn iid from regular \( D \))

- We give the item to bidder \( i^* = \arg \max_i \phi(v_i) \) when \( \phi(v_{i^*}) \geq 0 \).
What do revenue maximizing auctions look like? (when $v_i$ drawn iid from regular $D$)

▶ We give the item to bidder $i^* = \arg \max_i \phi(v_i)$ when $\phi(v_{i^*}) \geq 0$.

▶ Because $\phi$ is monotone, $i^* = \arg \max_i v_i$: the item goes to the highest bidder when $\phi(v_{i^*}) \geq 0$. 

Winner pays $v_{i^*} - Rv_{i^*}p^* = p^*$, where:

▶ i.e. it’s just a Vickrey auction with a reserve price of $\phi^{-1}(0)$!

▶ Remarkable — Simple eBay style auction is the best possible.
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- Winner pays \( v_{i^*} - \int_{p^*}^{v_{i^*}} 1 = p^* \), where:

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p^* = \max \left( \max_{i \neq i^*} v_i, \phi^{-1}(0) \right)
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Myserson Optimal Auctions

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  - Each bidder has their own virtual valuation function $\phi_i(v_i)$.
  - Auction no longer so natural. e.g. high bidder no longer necessarily wins.

- Doesn’t extend beyond single parameter domains...
- Requires knowledge of $D$...
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One more thing

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- If we care about revenue, should we give up on welfare?
- The Vickrey auction yields no revenue selling to a single bidder, whereas when $D$ is uniform over $[0, 1]$ we can get expected revenue $1/4$.
- What about a Vickrey auction with 2 bidders?

$$\text{Rev(VA)} = E_{v_1, v_2 \sim D} \left[ \min(v_1, v_2) \right] = 1/3$$
If we care about revenue, should we give up on welfare?

The Vickrey auction yields *no* revenue selling to a single bidder, whereas when $D$ is uniform over $[0, 1]$ we can get expected revenue $1/4$.

What about a Vickrey auction with 2 bidders?

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Rev(\text{VA}) = E_{v_1, v_2 \sim D}[\min(v_1, v_2)] = 1/3
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So we might be better off maximizing welfare with more bidders...
The Bulow/Klemperer Theorem

Theorem
Consider bidders drawn i.i.d. from a regular distribution $D$. For any $n \geq 1$, the Vickrey auction with $n + 1$ bidders has higher expected revenue than the revenue optimal auction with $n$ bidders.
The Bulow/Klemperer Theorem

**Theorem**
Consider bidders drawn i.i.d. from a regular distribution $D$. For any $n \geq 1$, the Vickrey auction with $n + 1$ bidders has higher expected revenue than the revenue optimal auction with $n$ bidders. So recruiting just one extra bidder is worth more than optimizing revenue for the current population.
The Bulow/Klemperer Theorem

Consider the hypothetical auction $A$ for $n + 1$ bidders:

1. Run the revenue optimal auction for the first $n$ bidders.
The Bulow/Klemperer Theorem

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1. Run the revenue optimal auction for the first $n$ bidders.
2. If the auction fails to allocate the item, give it to bidder $n + 1$ for free.

**Observations:**

1. The revenue of $A$ is exactly equal to the optimal revenue obtainable from $n$ bidders.
2. $A$ always allocates the item.
But...

- **Claim**: The Vickrey mechanism is obtains the maximum revenue amongst all mechanisms that always allocate the item.
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- Recall that $E_v[\sum_i P_i(v)] = E[\sum_i \phi_i(v_i) \cdot X_i(v)]$. 


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▸ **Claim**: The Vickrey mechanism is obtains the maximum revenue amongst all mechanisms that always allocate the item.

▸ Recall that $E_v[\sum_i P_i(v)] = E[\sum_i \phi_i(v_i) \cdot X_i(v)]$.

▸ We can maximize the RHS (subject to always allocating the item) by always allocating to $\arg \max_i \phi(v_i)$.
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- Recall that $E_v[\sum_i P_i(v)] = E[\sum_i \phi_i(v_i) \cdot X_i(v)]$.
- We can maximize the RHS (subject to always allocating the item) by always allocating to $\text{arg max}_i \phi(v_i)$.
- Since $D$ is regular, $\phi$ is monotone: this is $\text{arg max}_i v_i$ — the Vickrey allocation!
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- Recall that $E_v[\sum_i P_i(v)] = E[\sum_i \phi_i(v_i) \cdot X_i(v)]$.

- We can maximize the RHS (subject to always allocating the item) by always allocating to $\arg \max_i \phi(v_i)$.

- Since $D$ is regular, $\phi$ is monotone: this is $\arg \max_i v_i$ — the Vickrey allocation!

- So: The Vickrey-auction with $n+1$ bidders has only higher revenue than the optimal $n$ bidder auction.
Thanks!

See you next class — stay healthy!