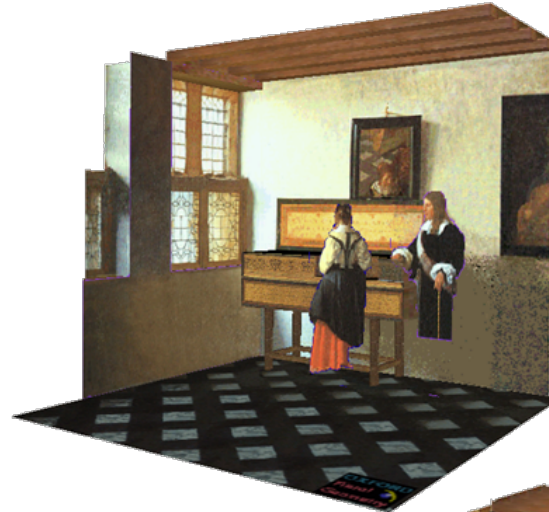# Single View Metrology

Slides taken from Steve Seitz, A. Efros

Applications of projective geometry
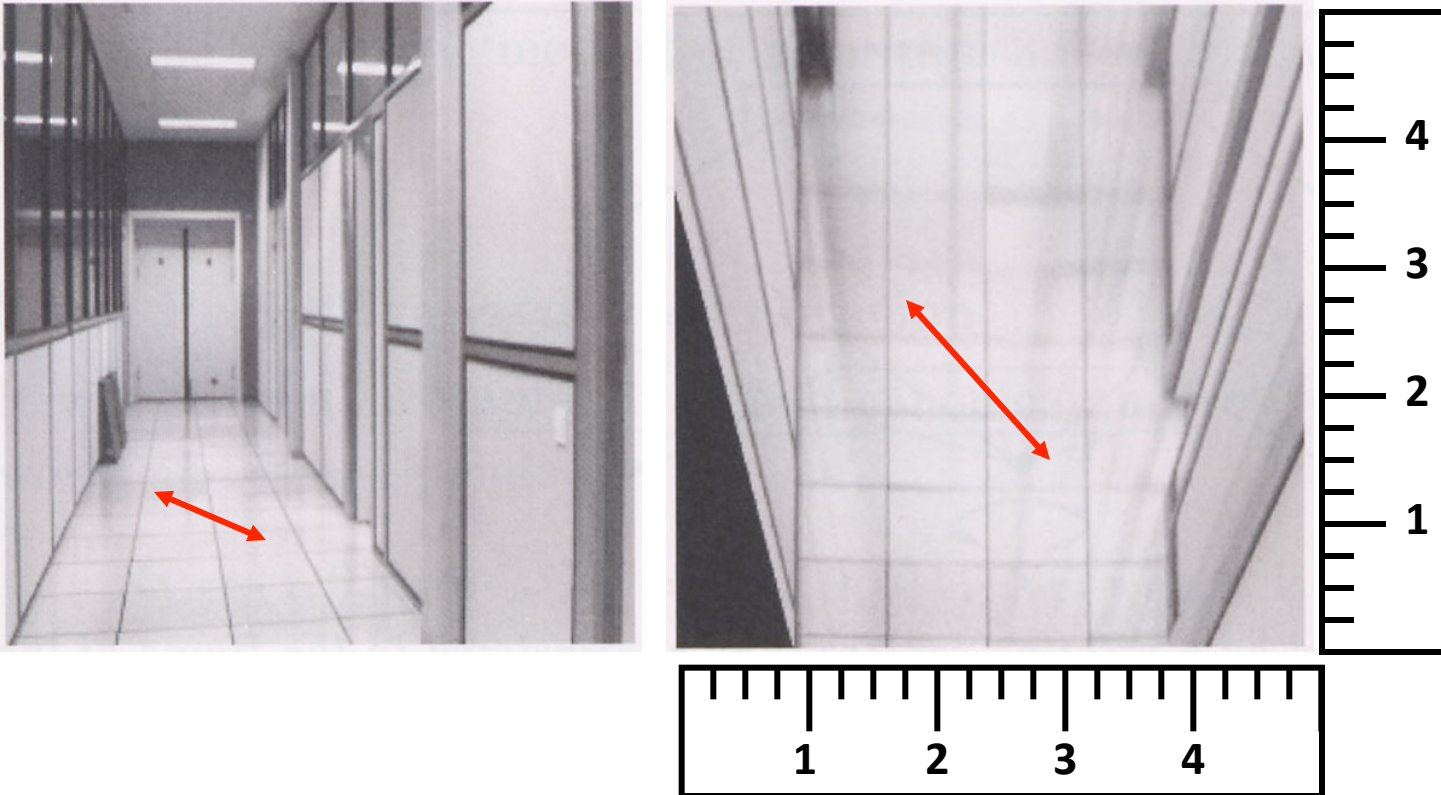

Vermeer's *Music Lesson*

- Criminisi et al., "Single View Metrology", ICCV 1999
- Other methods
  - Horry et al., "Tour Into the Picture", SIGGRAPH 96
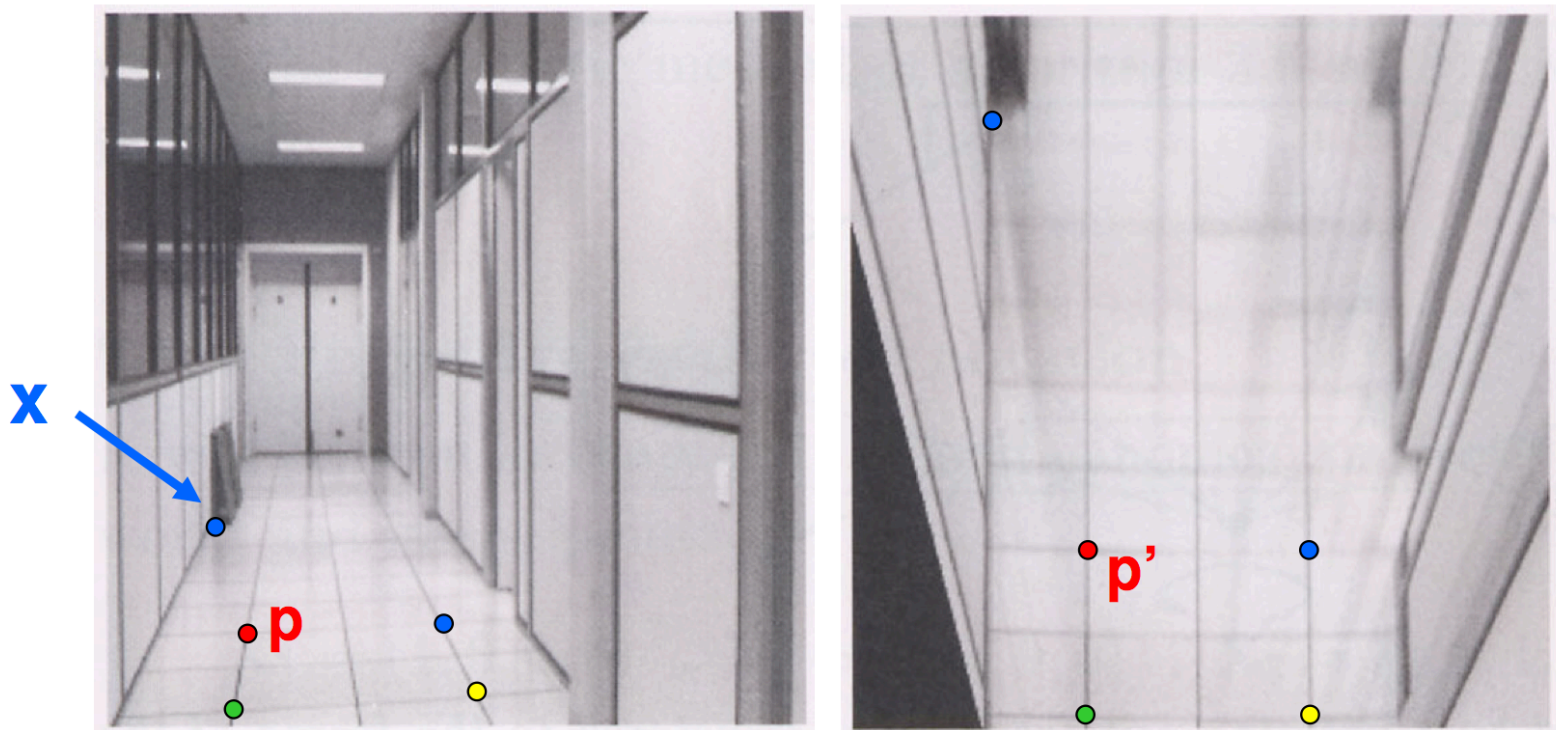 - Shum et al., CVPR 98
  - …

Measurements on planes

Approach: unwarp then measure

What kind of warp is this?

- A *Homography*
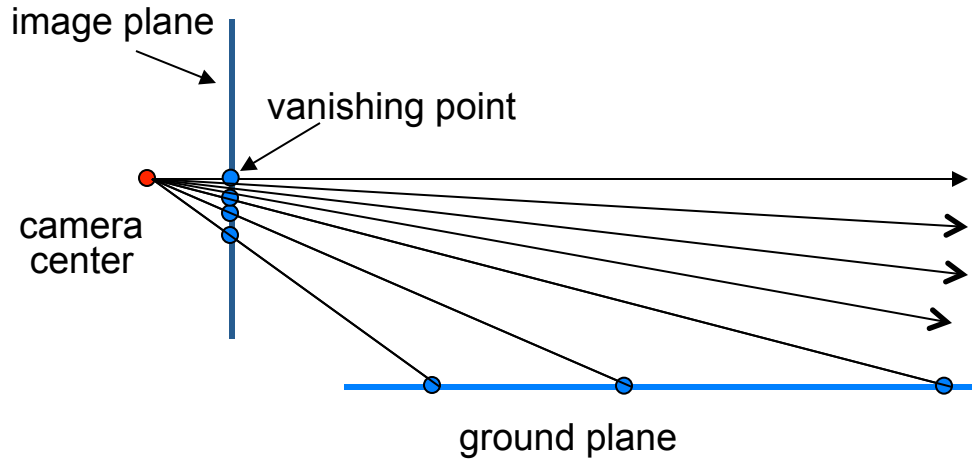
Image rectification, and measurement

Our old friend – the homography

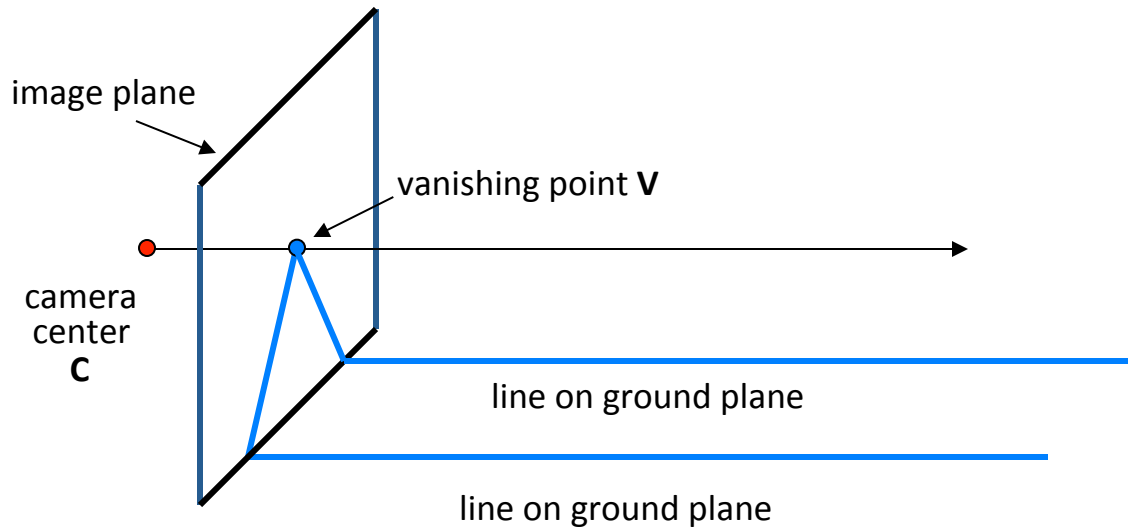Need 4 reference points with world coordinates

$$p = (x, y)$$
$$p' = (X, Y, 0)$$

Vanishing points

- Vanishing point
 - projection of a point at infinity

Vanishing points (2D)

image plane

vanishing point

camera
center

line on ground plane

Vanishing points

image plane

vanishing point **V**

camera
center
C

line on ground plane

line on ground plane

- Properties
  - Any two parallel lines have the same vanishing point
  - The ray from **C** through **v** point is parallel to the lines
  - An image may have more than one vanishing point

Vanishing lines
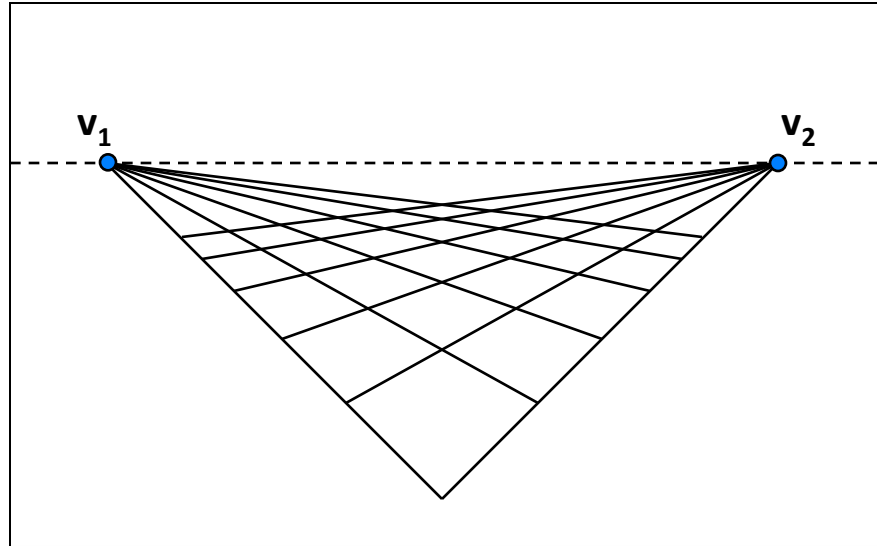
- Multiple Vanishing Points
  - Any set of parallel lines on the plane define a vanishing point
  - The union of all of these vanishing points is the *horizon line*
    - also called *vanishing line*
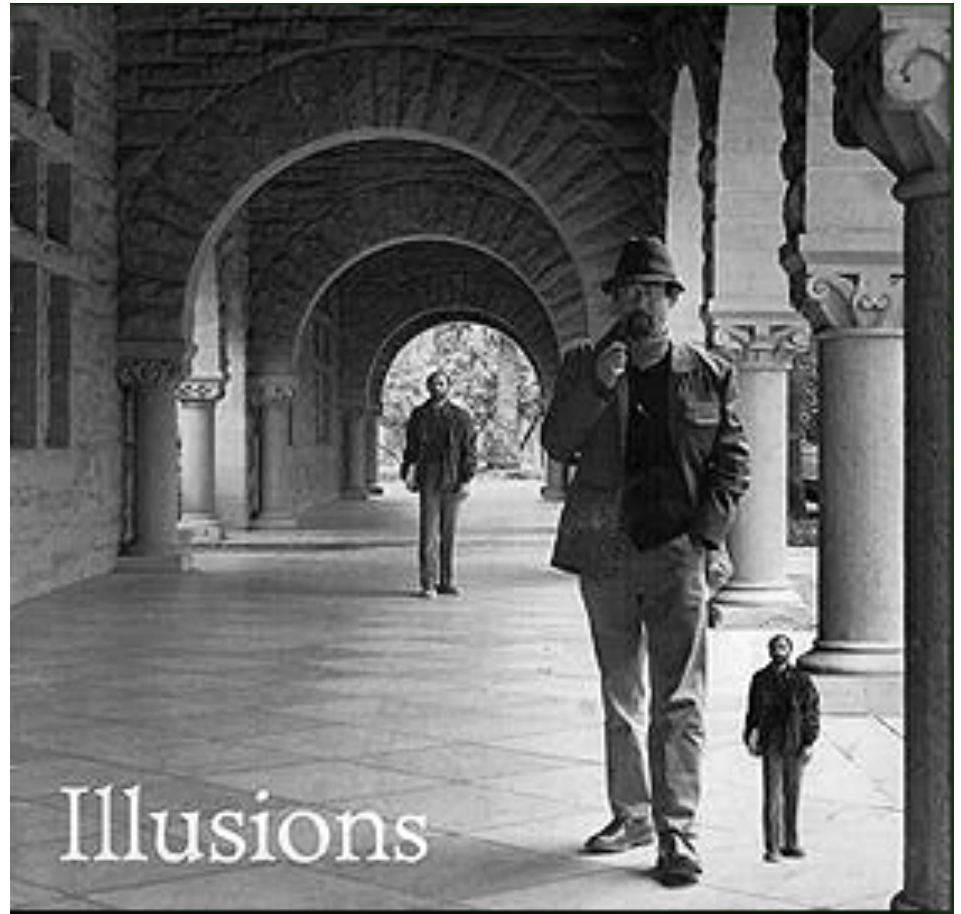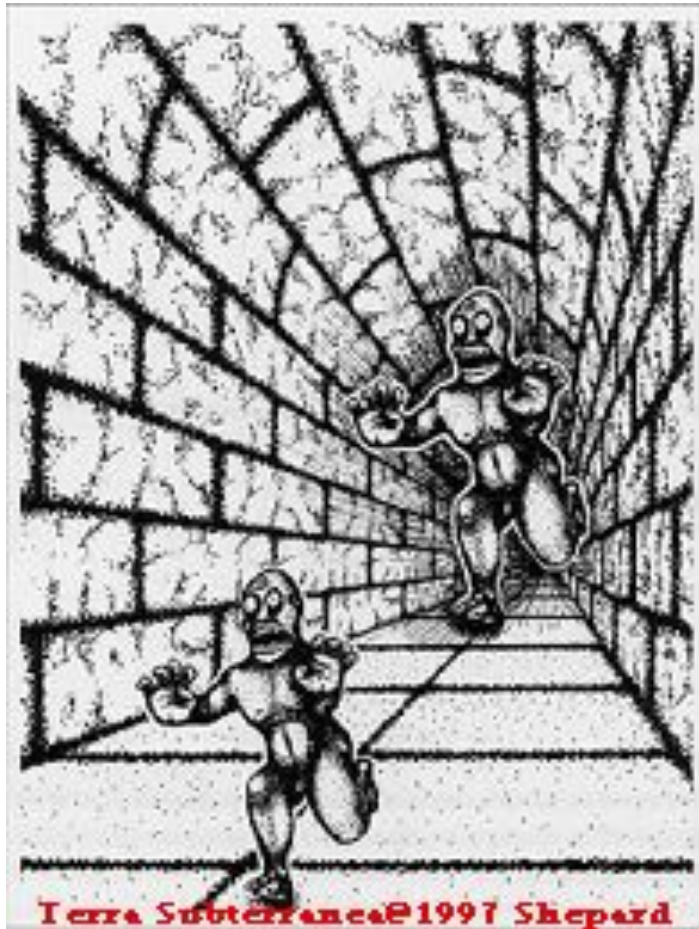 - Note that different planes define different vanishing lines
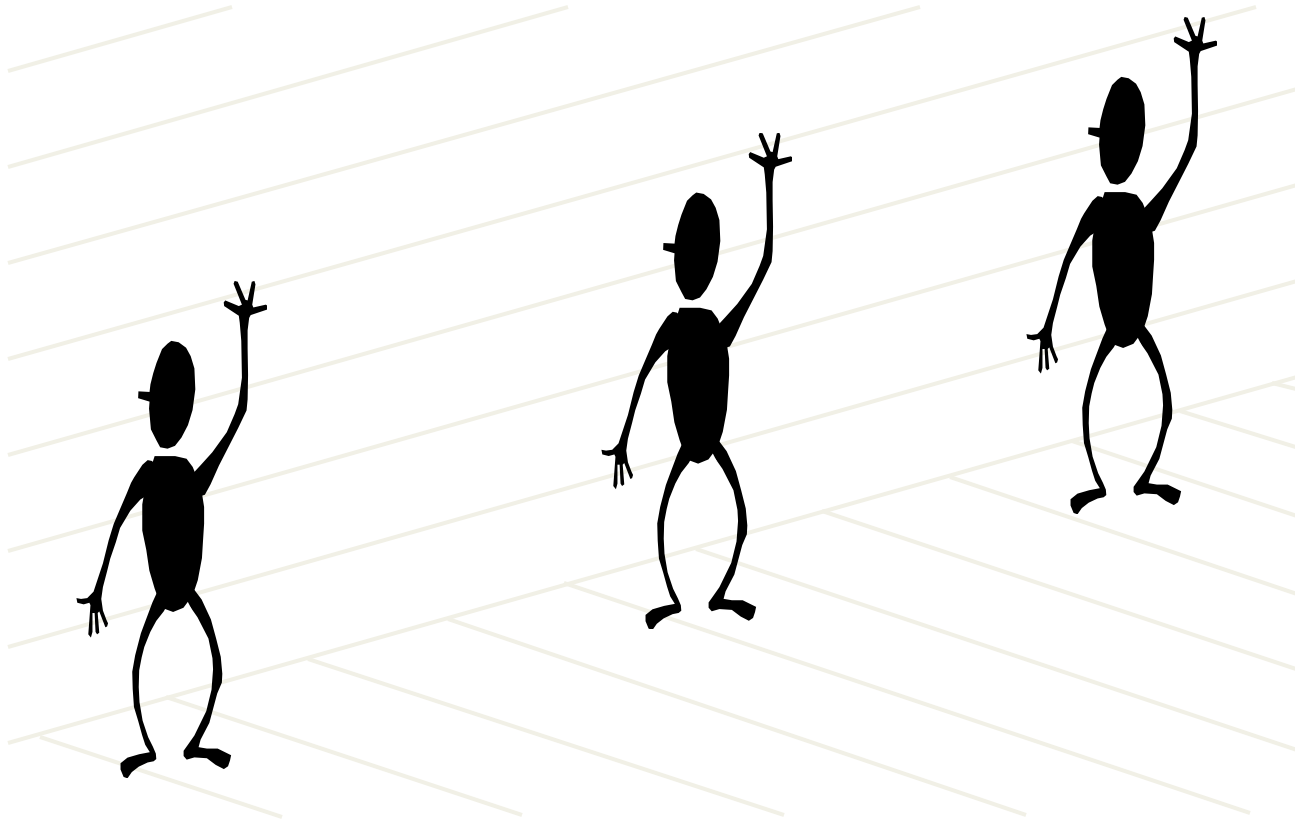
Computing vanishing lines
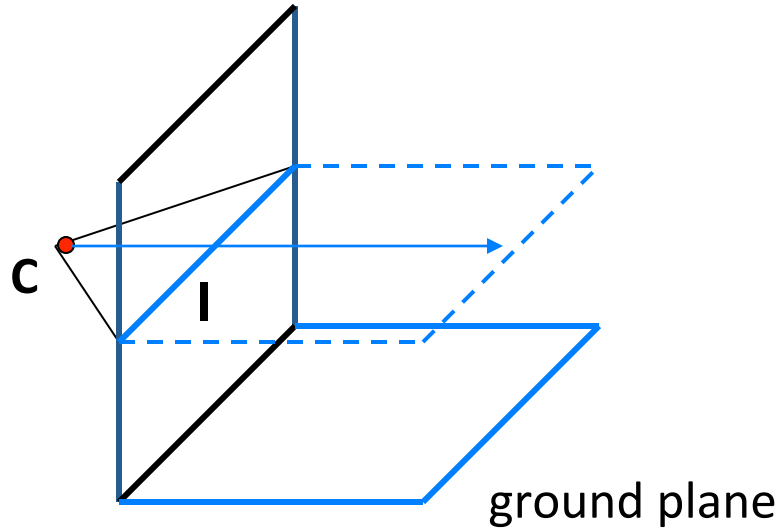
ground plane

- Properties
  - **l** is intersection of horizontal plane through **C** with image plane
  - Compute **l** from two sets of parallel lines on ground plane
  - All points at same height as **C** project to **l**
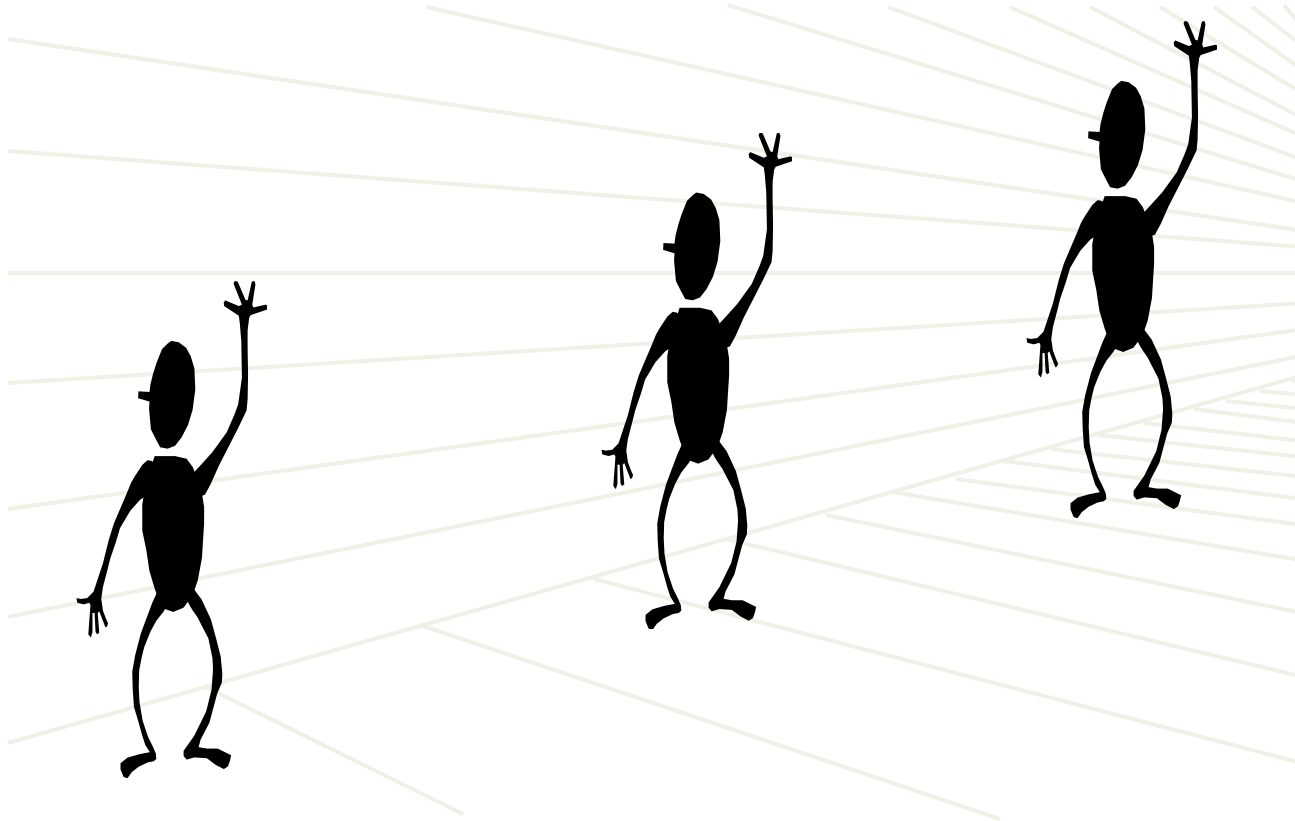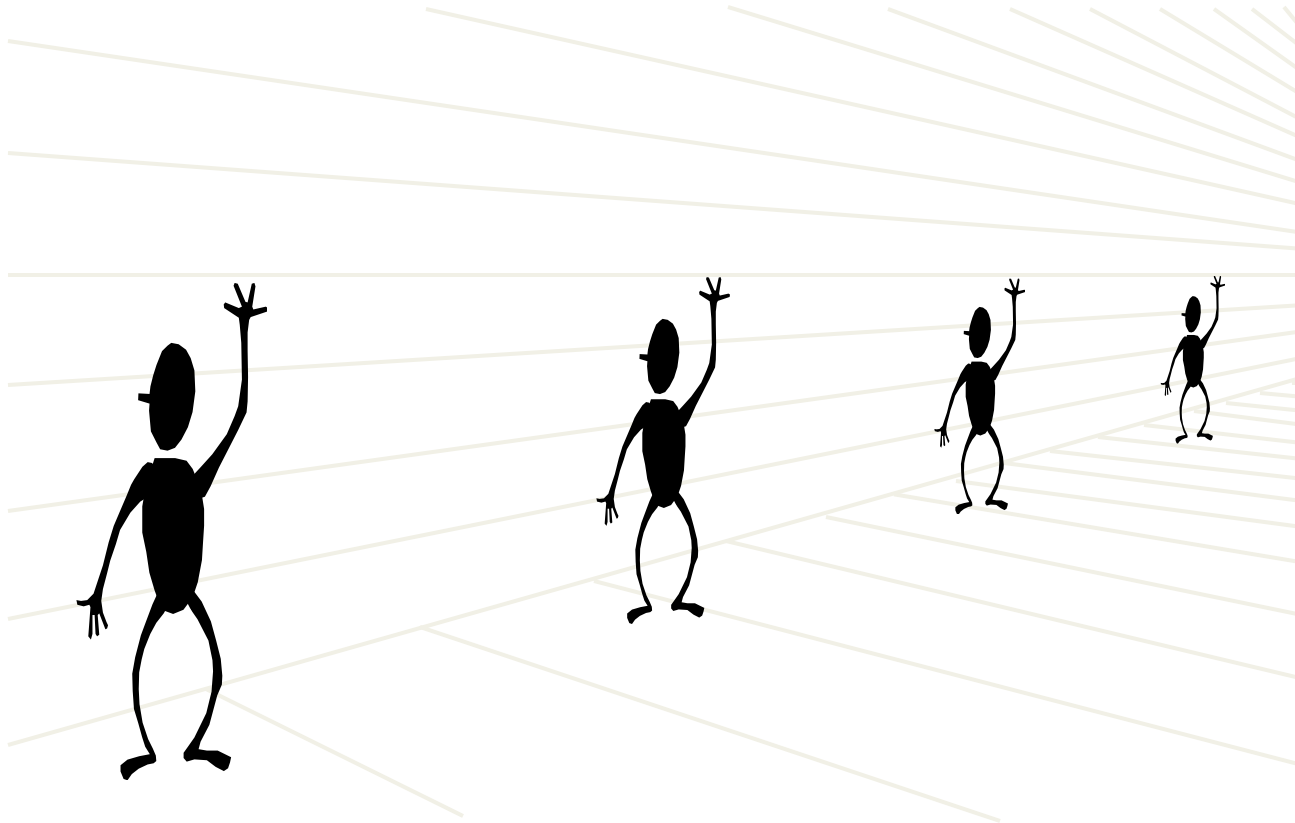 - Provides way of comparing height of objects in the scene
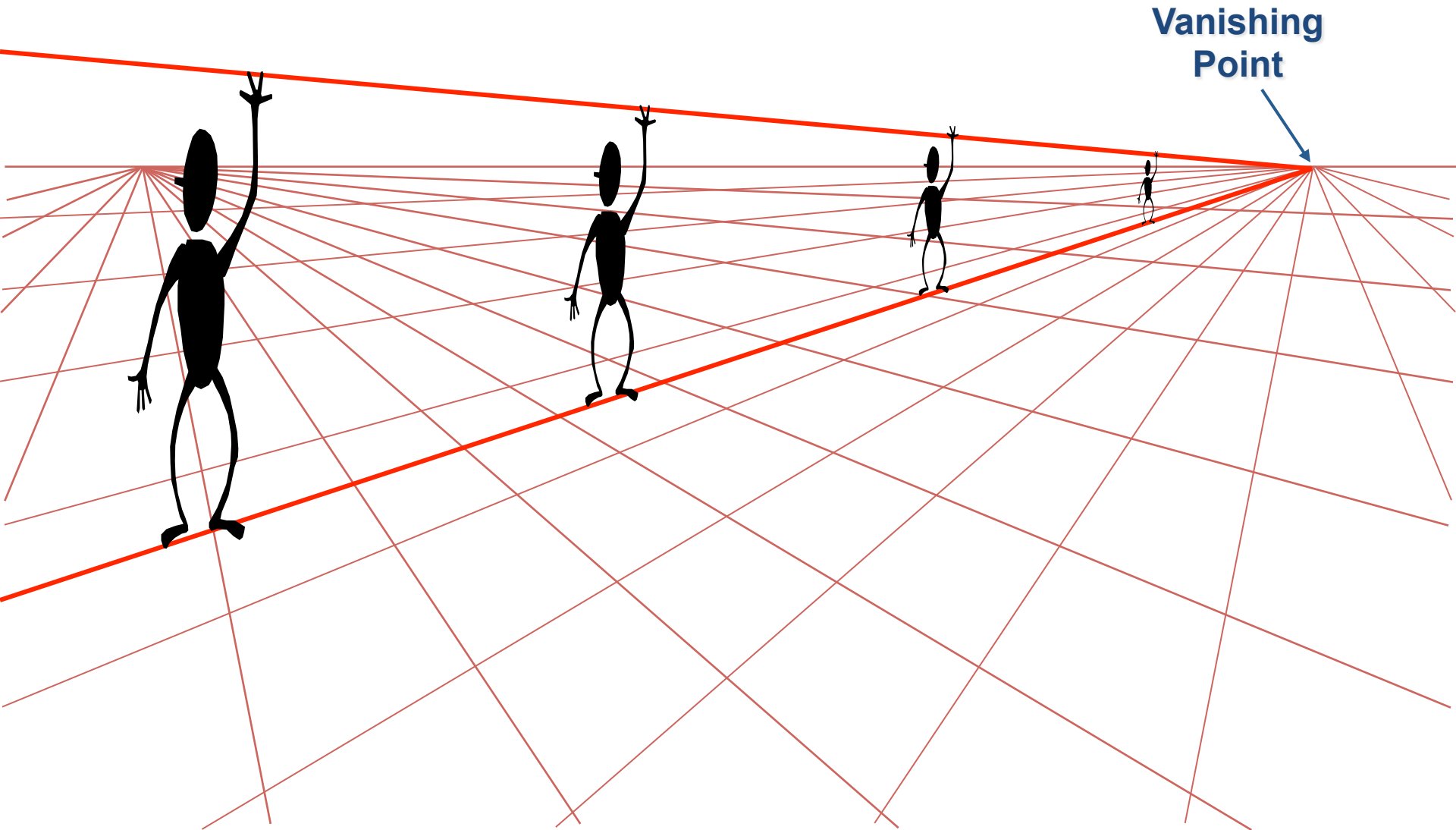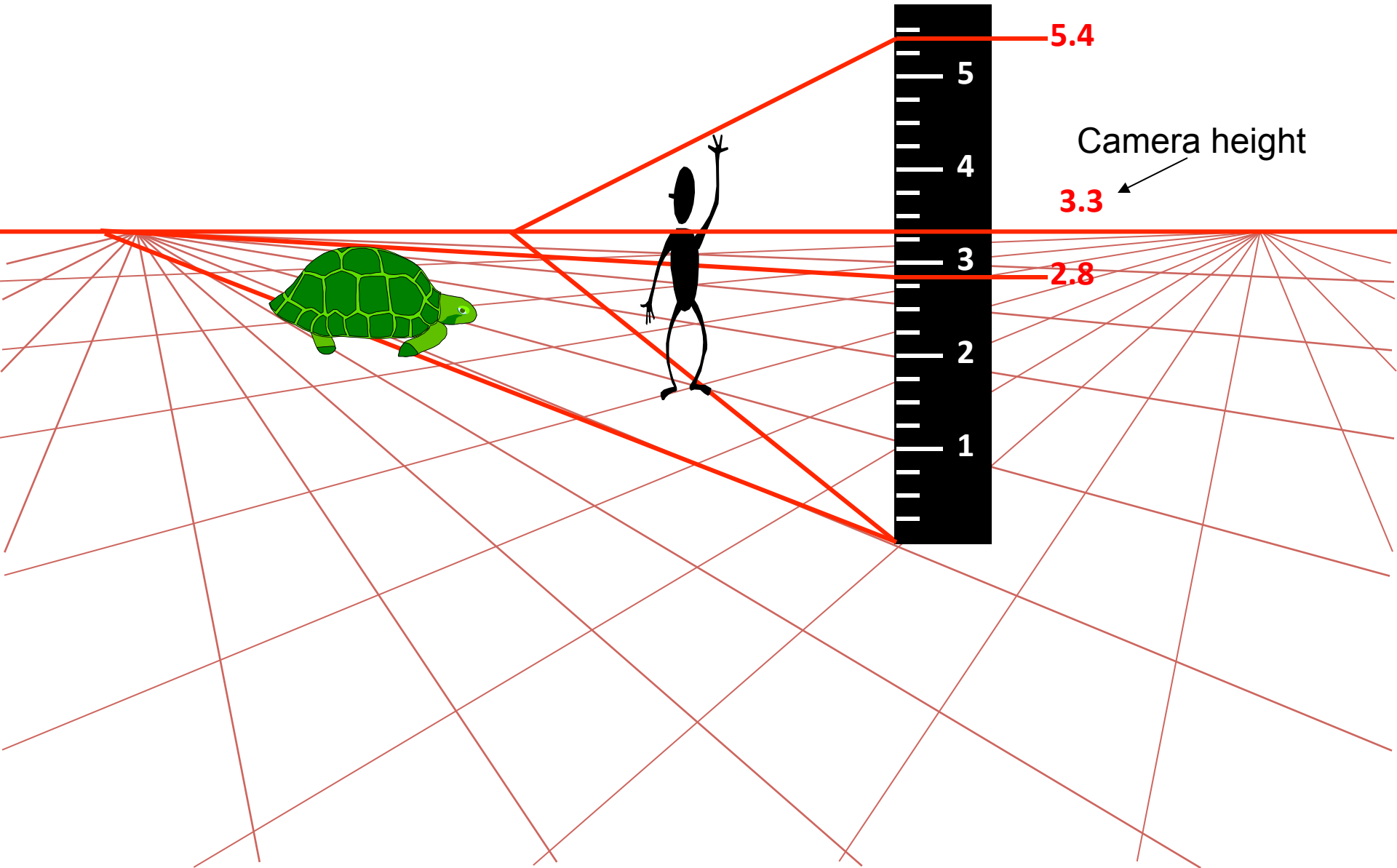
Fun with vanishing points

Perspective cues

Perspective cues

Perspective cues

Comparing heights

Vanishing Point

Measuring height

5.4

Camera height

3.3

5

4

3

2

1

2.8
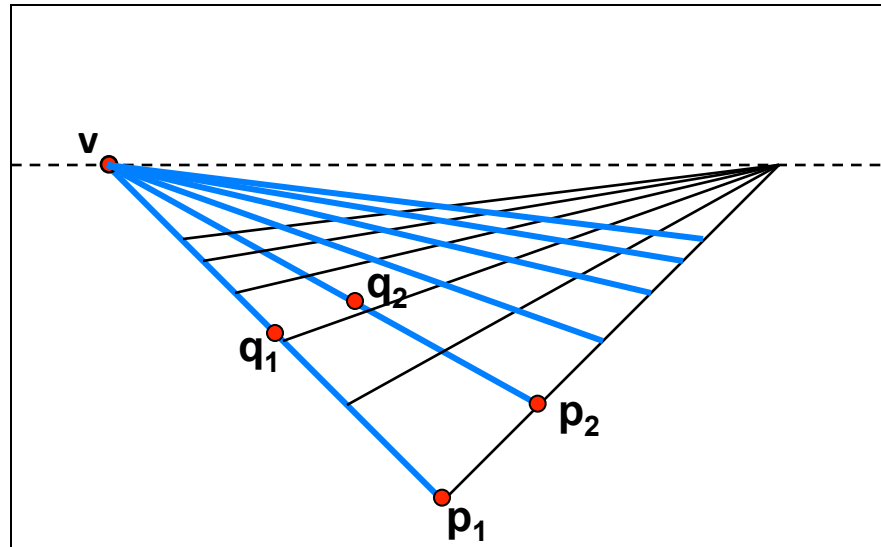
Computing vanishing points (from lines)

- Intersect p_1q_1 with p_2q_2

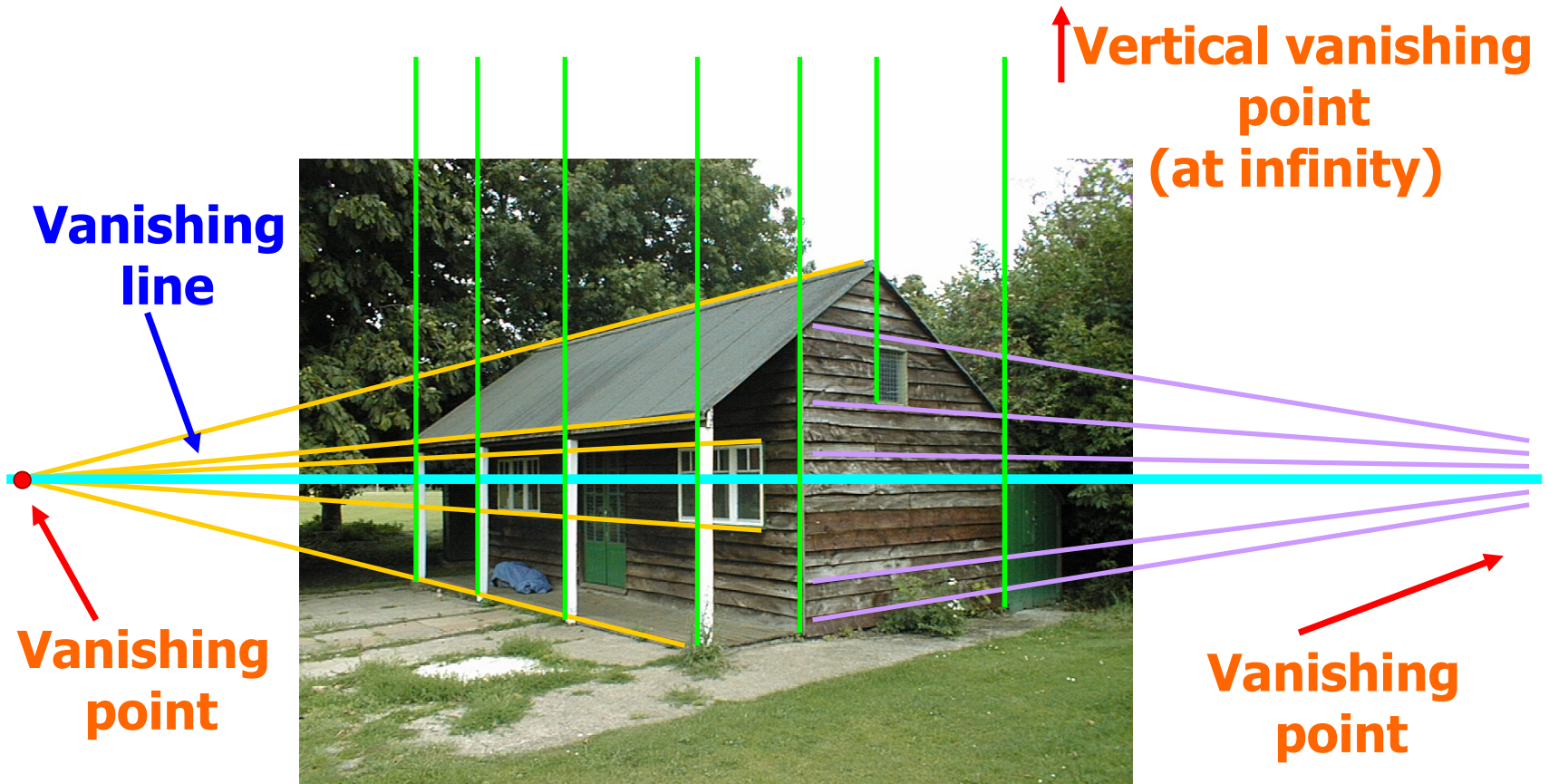$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

Least squares version

- Better to use more than two lines and compute the "closest" point of intersection
- See notes by Bob Collins for one good way of doing this:
  - http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt

Criminisi '99

Vertical vanishing point (at infinity)

Vanishing line

Vanishing point

Vanishing point

Measuring height without a ruler
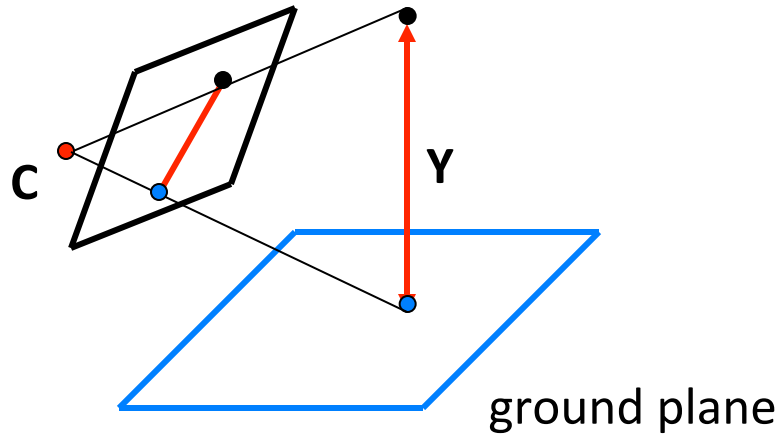
ground plane

Compute Y from image measurements

- Need more than vanishing points to do this

Measuring height

$\uparrow \mathbf{v_z}$

vanishing line (horizon)

$v \cong (b \times b_0) \times (v_x \times v_y)$
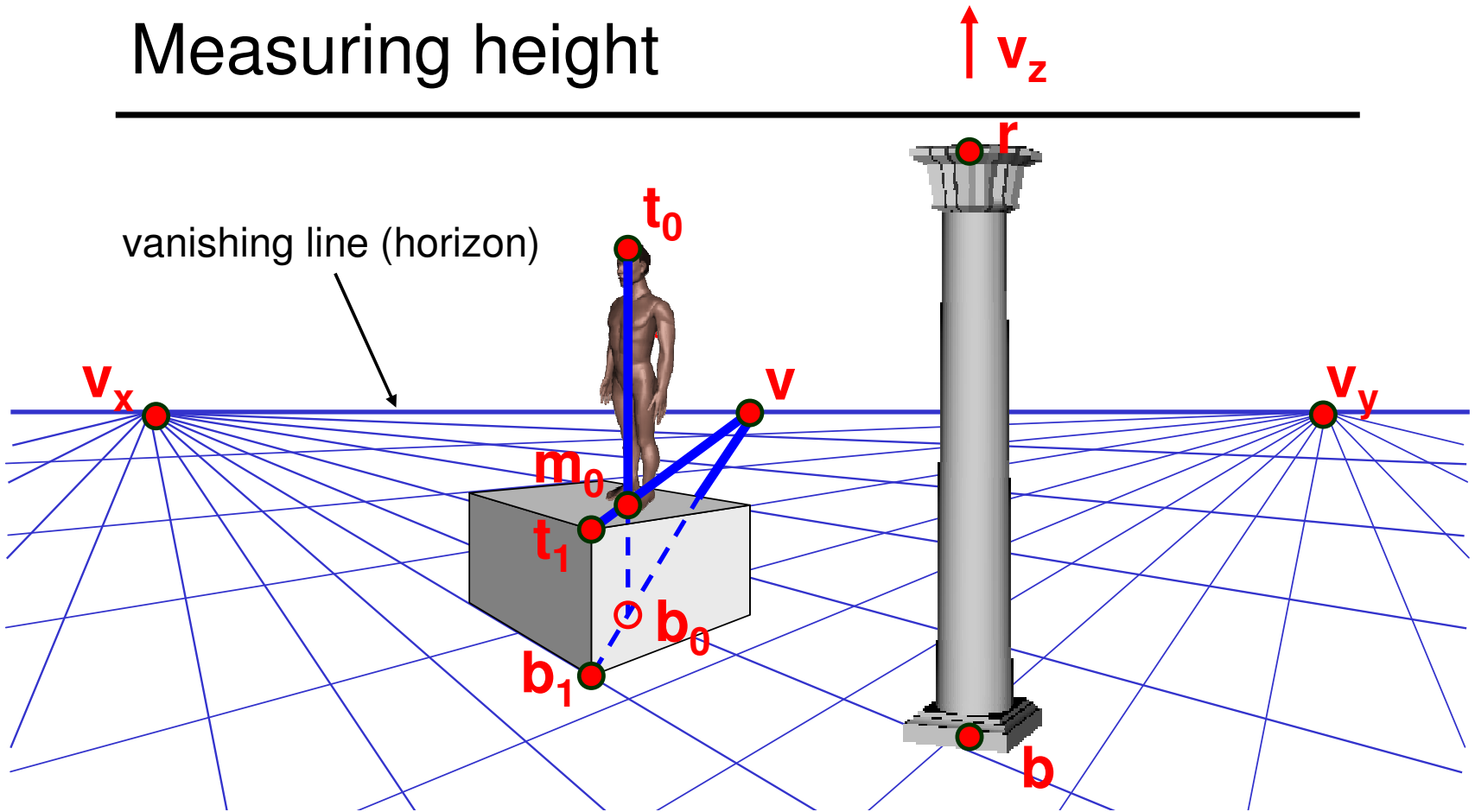
$t \cong (v \times t_0) \times (r \times b)$

$\mathbf{v_x}$ $\mathbf{v}$ $\mathbf{t_0}$ $\mathbf{t}$ $\mathbf{r}$ $\mathbf{v_y}$

$\mathbf{b_0}$ $\mathbf{b}$

H **R** **H**

Measuring height

v_z

r

vanishing line (horizon)

t_0

v_x

v

v_y

m_0

t_1

b_0

b_1

b
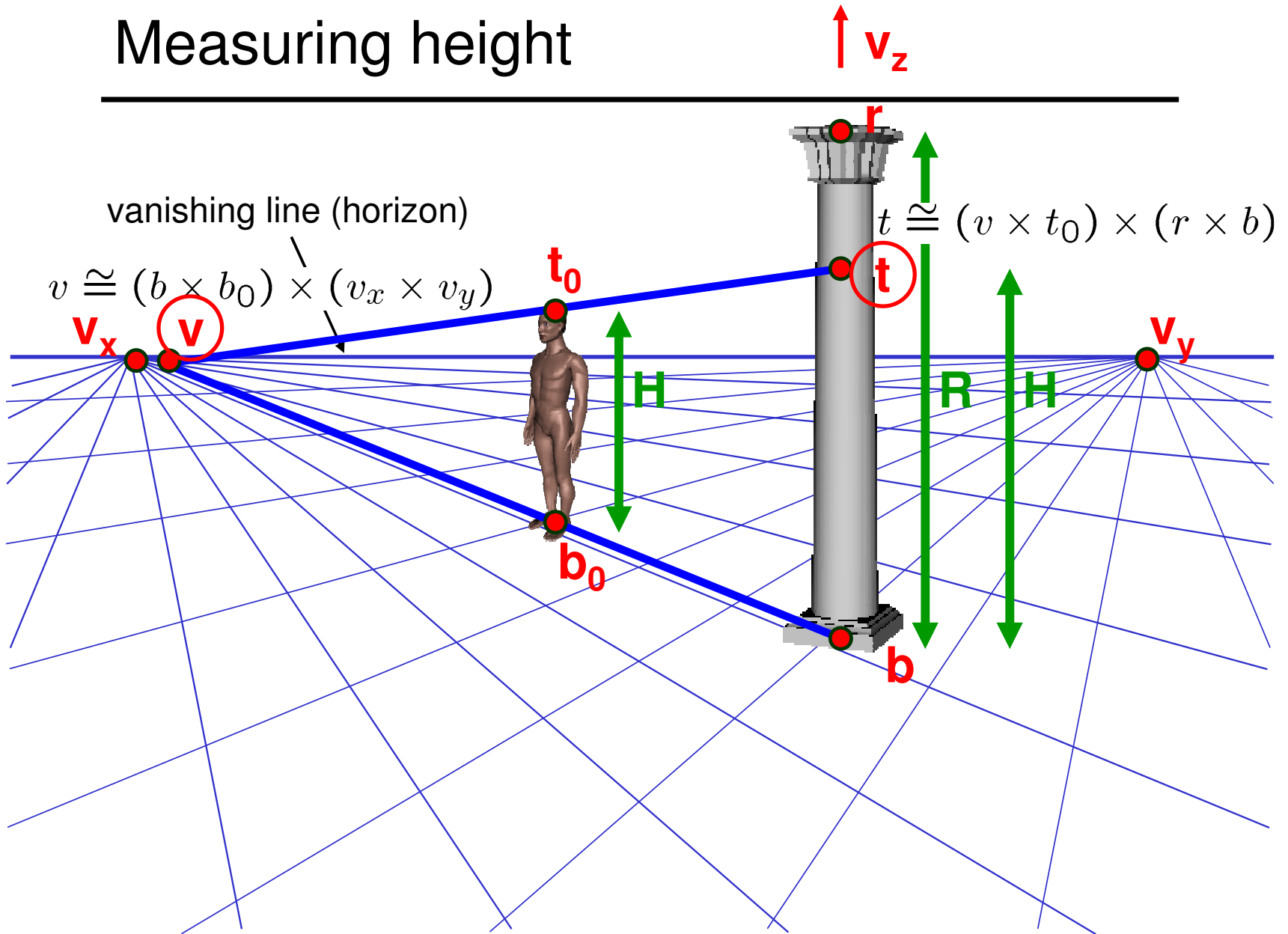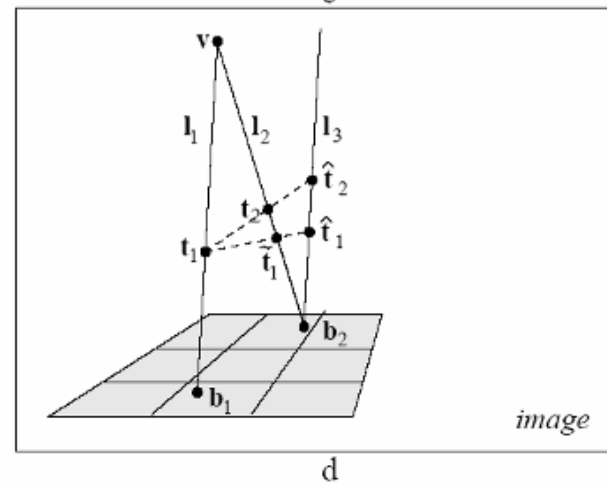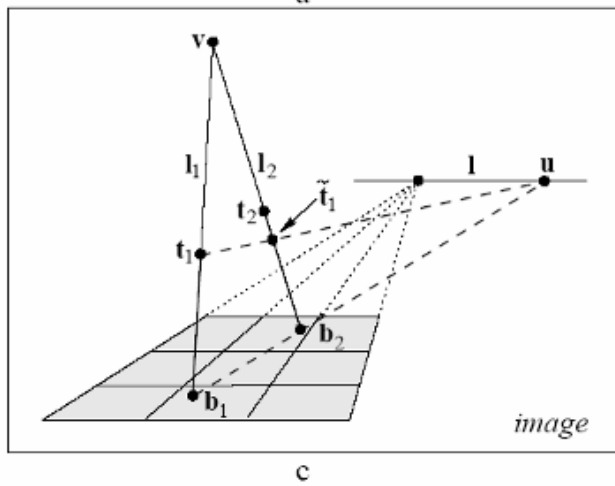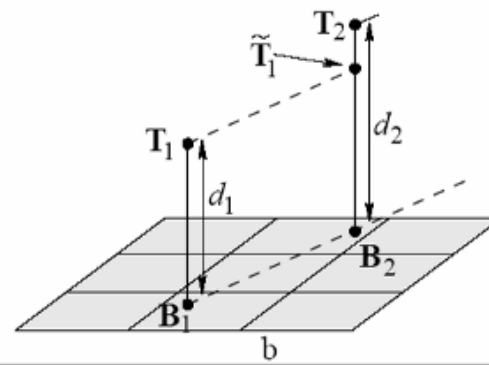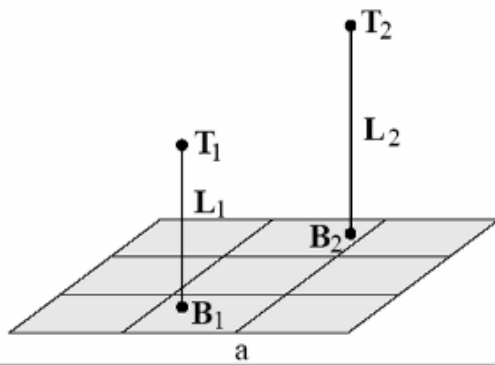
What if the point on the ground plane b_0 is not known?

- Here the guy is standing on the box
- Use one side of the box to help find b_0 as shown above

What if v_z is not infinity?
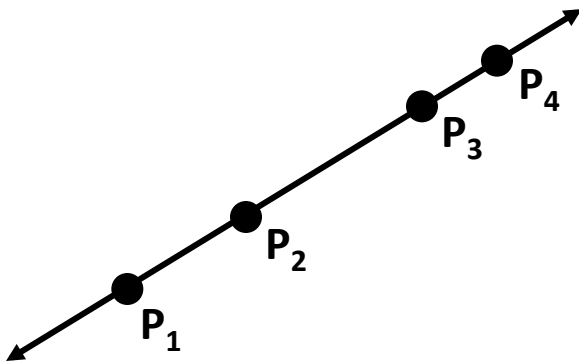
a

b

c

image

d

image

The cross ratio

- A Projective Invariant
  - Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points

$$\frac{\left\| \mathbf{P}_3 - \mathbf{P}_1 \right\| \left\| \mathbf{P}_4 - \mathbf{P}_2 \right\|}{\left\| \mathbf{P}_3 - \mathbf{P}_2 \right\| \left\| \mathbf{P}_4 - \mathbf{P}_1 \right\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Can permute the point ordering

$$\frac{\left\| \mathbf{P}_1 - \mathbf{P}_3 \right\| \left\| \mathbf{P}_4 - \mathbf{P}_2 \right\|}{\left\| \mathbf{P}_1 - \mathbf{P}_2 \right\| \left\| \mathbf{P}_4 - \mathbf{P}_3 \right\|}$$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height

∞

T (top of object)

R (reference point)

H

R

b

t

r

C

v$_z$

B (bottom of object)

ground plane
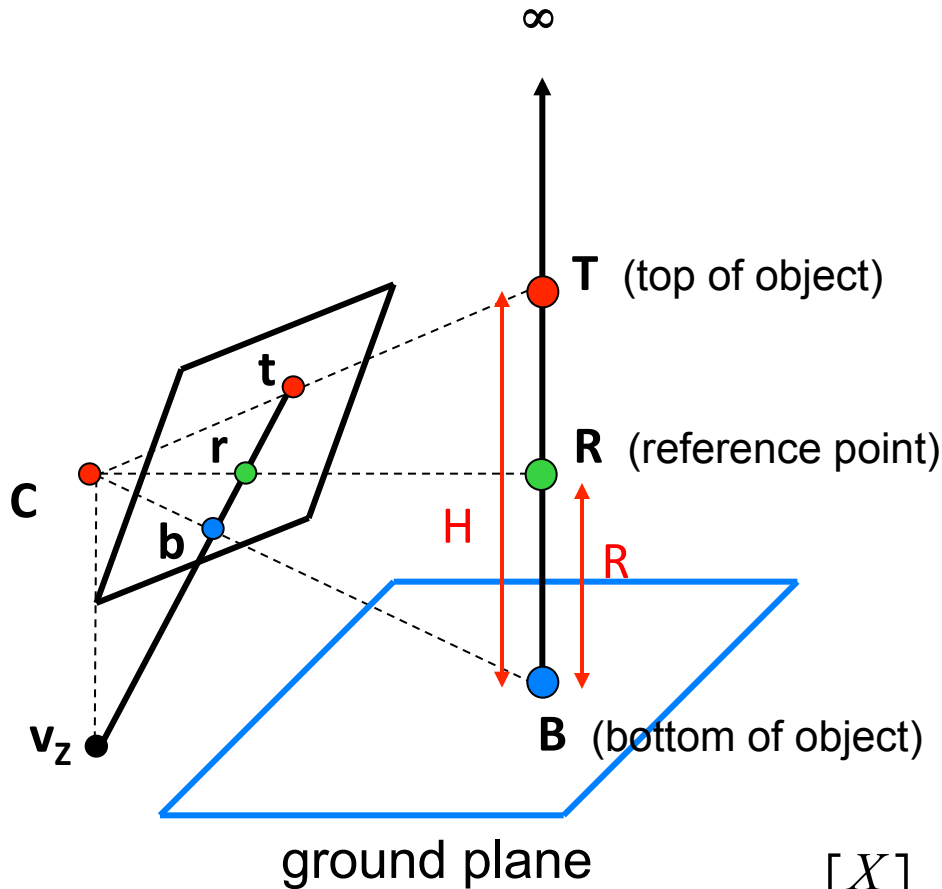
$$\frac{\|\mathbf{T}-\mathbf{B}\|\;\|\infty-\mathbf{R}\|}{\|\mathbf{R}-\mathbf{B}\|\;\|\infty-\mathbf{T}\|} = \frac{H}{R}$$
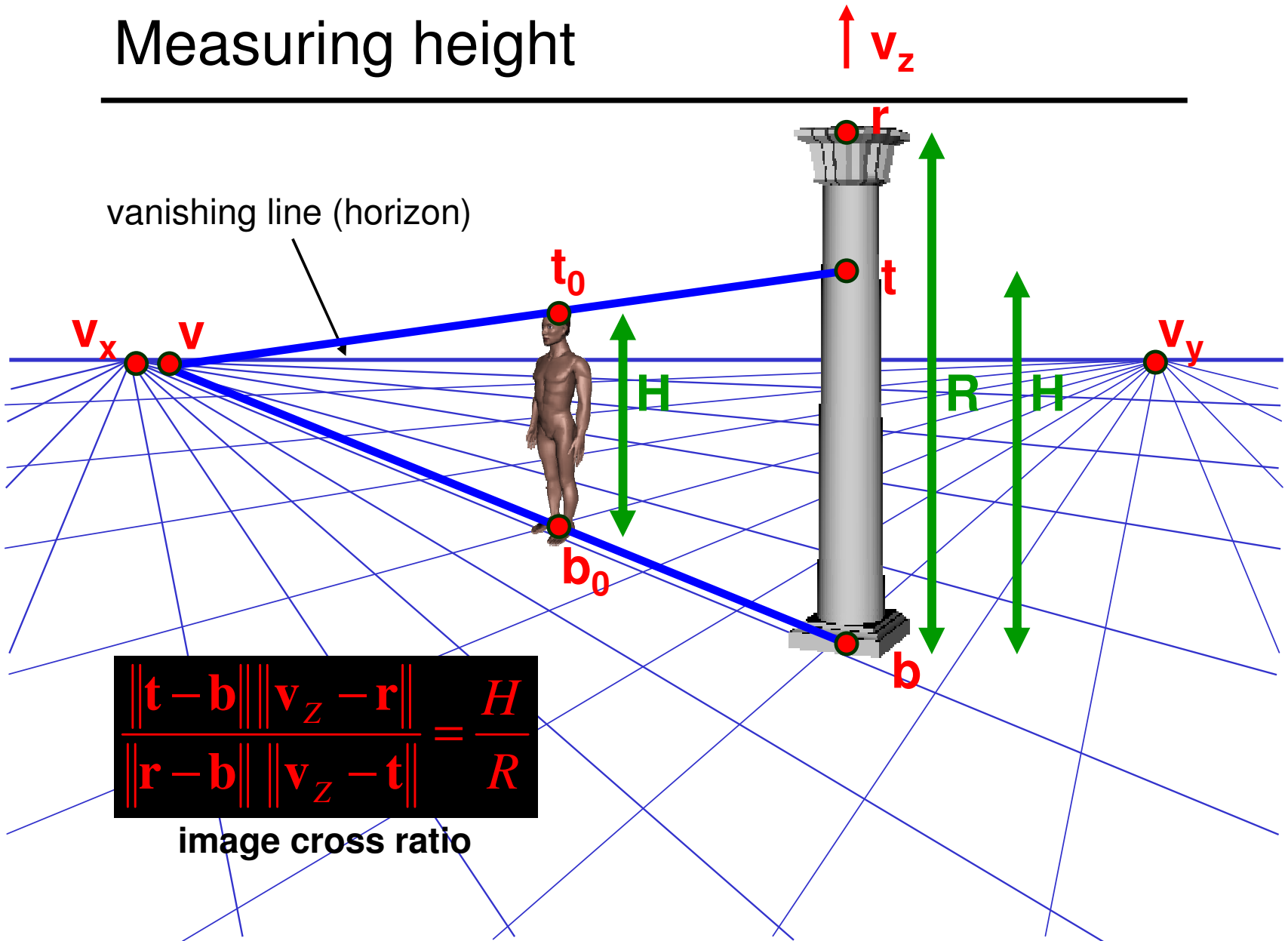
scene cross ratio

$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\;\|\mathbf{v}_Z-\mathbf{t}\|} = \frac{H}{R}$$

image cross ratio

scene points represented as $\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ image points as $\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
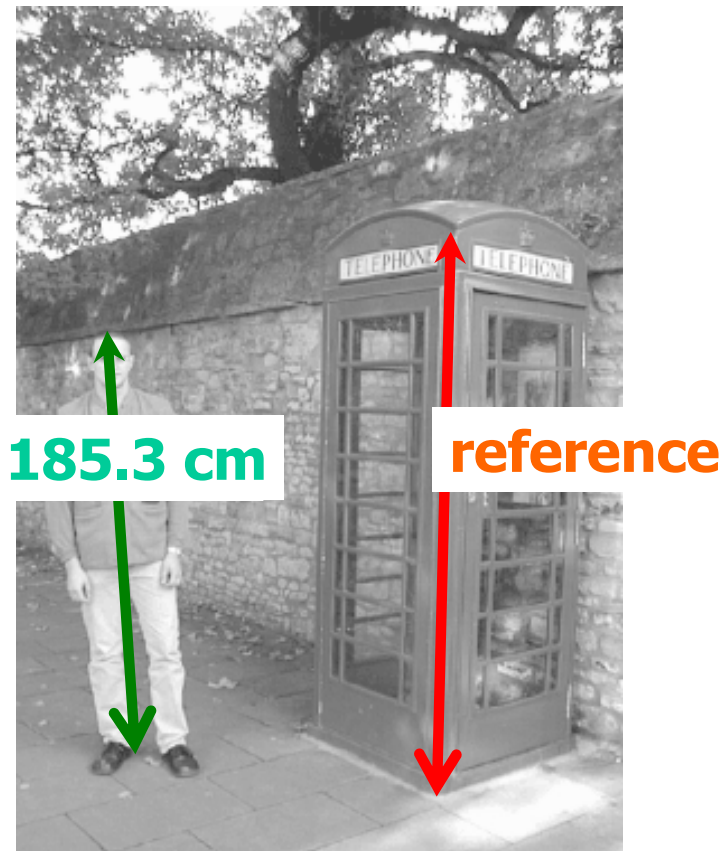
Measuring height

$\mathbf{v_z}$

r

vanishing line (horizon)

$\mathbf{t_0}$

t

$\mathbf{v_x}$ v

$\mathbf{v_y}$

H

R   H

$\mathbf{b_0}$

b

$$\frac{\|\mathbf{t} - \mathbf{b}\| \, \|\mathbf{v}_Z - \mathbf{r}\|}{\|\mathbf{r} - \mathbf{b}\| \, \|\mathbf{v}_Z - \mathbf{t}\|} = \frac{H}{R}$$

image cross ratio

Measuring heights of people

185.3 cm

reference

Here we go !

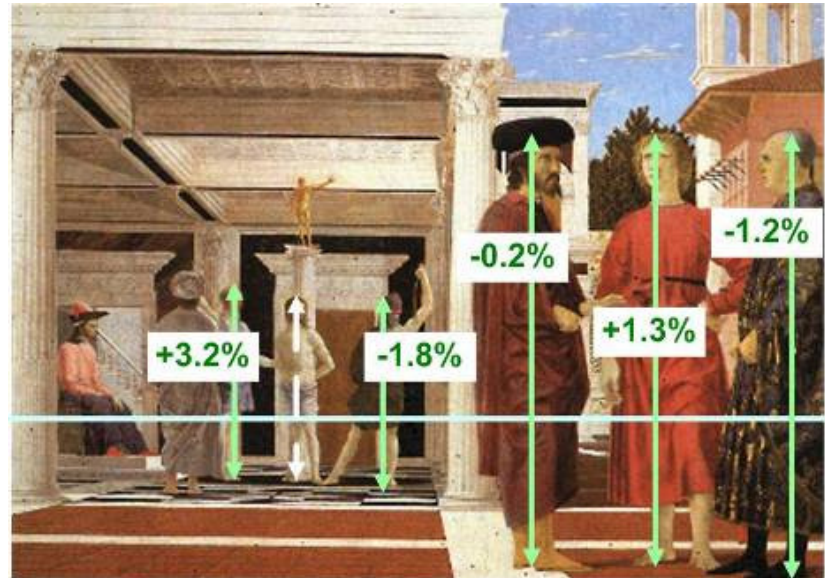# Forensic Science: measuring heights of suspects
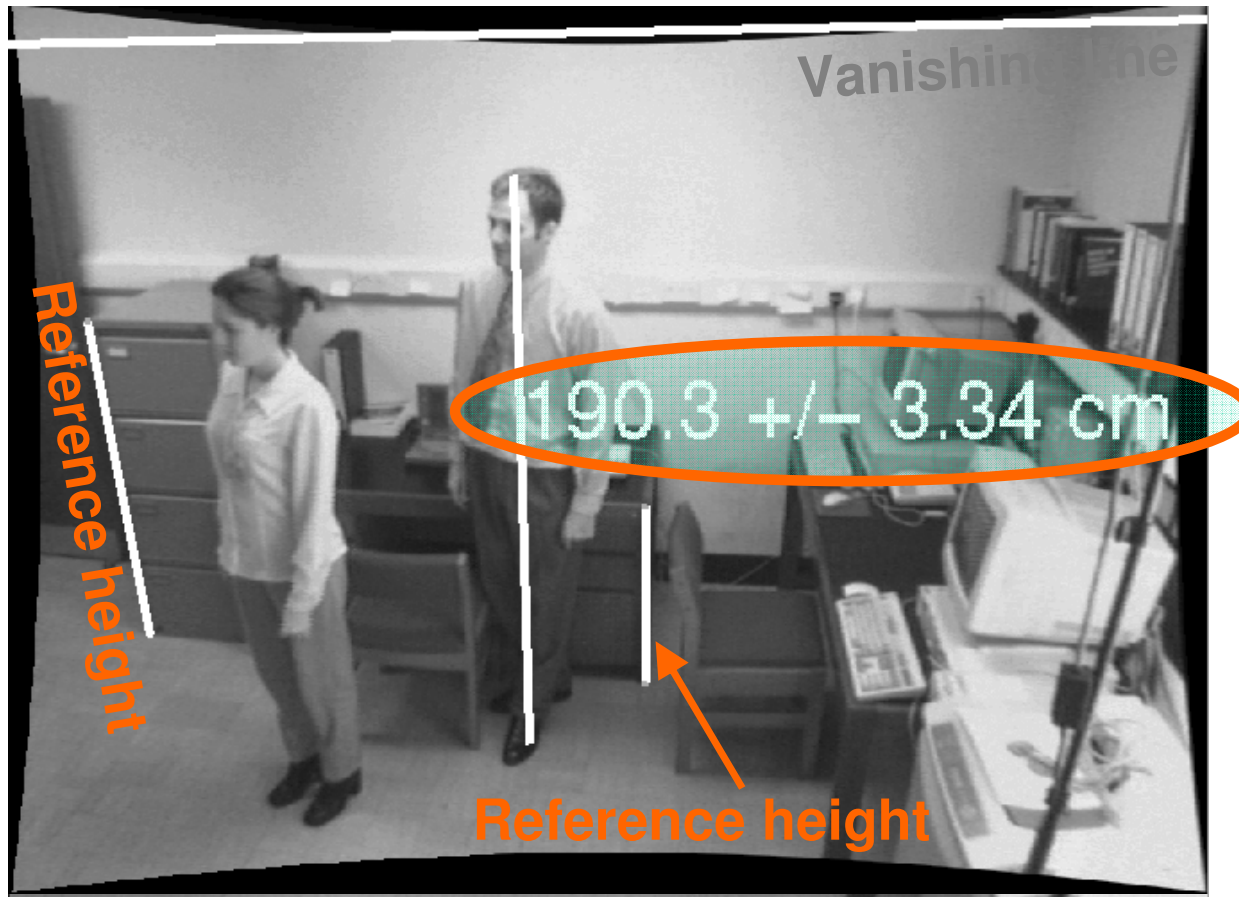
Assessing geometric accuracy

Are the heights of the 2 groups of people consistent with each other?

Flagellation,
Piero della Francesca

Estimated relative heights