1. Let $S_1$ be the following schema.

$$(\forall x)\neg Lxx \land (\forall x)(\forall y)(Lxy \supset Lyx) \land (\forall x)(\exists y)(\exists z)(y \neq z \land (\forall w)(Lxw \equiv (w = y \lor w = z)))$$

(a) (10 points) Specify a structure $A_1$ of size at least 6 which satisfies $S_1$, that is, $U^{A_1}$ has at least 6 members and $A_1 \models S_1$.

$U^{A_1} =$

$L^{A_1} =$

(b) (10 points) How many structures with universe of discourse \{1, 2, 3, 4, 5, 6\} satisfy $S_1$?

2. Let $S_2$ be the following schema.

$$(\forall x)(\forall y)(\forall z)((Lxy \land Lxz) \supset y = z) \land (\forall x)(\forall y)(\forall z)((Lxz \land Lyz) \supset x = y) \land (\forall x)(\exists y)Lxy$$

(a) (10 points) Specify a structure $A_2$ of size at least 6 which satisfies $S_2$.

$U^{A_2} =$

$L^{A_2} =$

(b) (10 points) How many structures with universe of discourse \{1, 2, 3, 4, 5, 6\} satisfy $S_2$?
3. Let $S_3$ be the conjunction of the following two schema.

- $(\forall x)(\forall y)(\forall z)(((Lxy \land Lxz) \supset y = z) \land (\forall y)Lxy$
- $(\forall x)(\forall y)(\forall z)(((Lxz \land Lyz) \supset x = y) \land (\exists y)(\exists x)\neg Lxy$

(a) (10 points) Specify a structure $A_3$ of size at least 3 which satisfies $S_3$.

$U^{A_3} =$

$L^{A_3} =$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy $S_3$?

4. Let $S_4$ be the following schema.

$(\forall x)(\forall y) Lxy \supset \neg Lxy \land (\forall x)(\forall y)(Lxy \lor Lyx \lor x = y)$

(a) (10 points) Specify a structure $A_4$ of size at least 3 which satisfies $S_4$.

$U^{A_4} =$

$L^{A_4} =$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy $S_4$?
5. Let $S_5$ be the conjunction of the following four schemata.

- $(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Rvwz \land Rxyz) \supset (v = x \land w = y))$
- $(\forall x)(\forall y)(\forall z)(Rxyz \supset (Fx \land Fy))$
- $(\forall x)(\forall y)((Fx \land Fy) \supset (\exists z)(\forall w)(Rxyw \equiv w = z))$
- $(\forall z)(\exists x)(\exists y)Rxyz$

(a) (10 points) Specify a structure $A_5$ of size at least 3 which satisfies $S_5$.

$U^{A_5} =$

$F^{A_5} =$

$R^{A_5} =$

(b) (10 points) How many structures with universe of discourse \{1, 2, 3, 4\} satisfy $S_5$?