For the purposes of this problem set, we restrict attention to pure monadic quantificational schemata all of whose predicate letters are among $F$ and $G$, and to structures which interpret exactly these predicate letters. We employ the following terminology in the problems below.

- A list of pure monadic schemata is *succinct* if and only if no two schemata on the list are equivalent.
- A pure monadic schema *implies a list of schemata* if and only if it implies every schema on the list.
- The *power* of a pure monadic schema is the length of a longest succinct list of pure monadic schemata it implies.
- A pure monadic schema *rules* a number $n$ if and only if more than half of the structures with universe of discourse $\{1, \ldots, n\}$ satisfy $S$.

1. (25 points) What is the length of a longest list of schemata no two of which have the same power?

2. (25 points) What is the length of a longest succinct list of schemata all of which have the same power?

3. (25 points) What is the maximum power of a schema that rules 4?

4. (25 points) What is the maximum power of a schema that rules 10?