For the purposes of this problem set, we restrict attention to monadic quantificational schemata (abbreviated MQ-schemata) all of whose predicate letters are among $F$ and $G$, and to structures which interpret exactly these predicate letters. We employ the following terminology in the problems below.

- If $S$ and $T$ are MQ-schemata we say that a structure $A$ is a counterexample to the claim that $S$ implies $T$ if and only if $A \models S$ and $A \not\models T$.

- If $S$ and $T$ are MQ-schemata we say that a structure $A$ witnesses the inequivalence of $S$ and $T$ if and only if either $A$ is a counterexample to the claim that $S$ implies $T$ or $A$ is a counterexample to the claim that $T$ implies $S$.

1. Let $S$ be the schema
   \[(\exists x)(Fx \land Gx) \land (\exists x)(Fx \land \neg Gx) \land (\exists x)(\neg Fx \land Gx) \land (\exists x)(\neg Fx \land \neg Gx)\]
   and let $T$ be the schema
   \[(\forall x)(Fx \equiv Gx).\]
   (a) (25 points) How many structures with universe of discourse $\{1, 2, 3\}$ are counterexamples to the claim that $S$ implies $T$?
   (b) (25 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ are counterexamples to the claim that $S$ implies $T$?

2. (25 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ witness the inequivalence of $(\forall x)(Fx \oplus Gx)$ and $(\forall x)(Fx \equiv Gx)$?

3. (25 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ witness the inequivalence of $(\exists x)(Fx \land Gx)$ and $(\forall x)(Fx \lor Gx)$?