1. We call a set of numbers $X$ good if and only if no member of $X$ evenly divides another member of $X$, that is, $X$ is good if and only if for all $i$ and $j$, if $i, j \in X$ and $i \neq j$, then for every $k$, $i \cdot k \neq j$.

(a) (10 points) What is the maximum size of a good set $X$ contained in $\{1, 2, \ldots, 100\}$?

(b) (15 points) Give an example of a maximum size good set $X \subseteq \{1, 2, \ldots, 100\}$ and explain why there is no larger such set.

2. (15 points) How many truth-assignments to the sentence letters $p_1, \ldots, p_5$ satisfy the following truth-functional schema?

$$(((p_1 \lor p_2) \lor p_3) \lor p_4) \lor p_5$$
3. For the purposes of this problem, we restrict attention to truth-functional schemata all of whose sentence letters are among \( p_1, p_2, p_3, \) and \( p_4 \). We employ the following terminology.

- A list of truth-functional schemata is *succinct* if and only if no two schemata on the list are equivalent.
- A truth-functional schema *implies a list of schemata* if and only if it implies every schema on the list.
- The *power* of a truth-functional schema is the length of a longest succinct list of schemata it implies.

(a) (15 points) What is the length of a longest succinct list of schemata, all of the same power, that all imply \(((p_1 \equiv p_2) \equiv p_3) \equiv p_4\)?

(b) (15 points) What is the largest number \( n \) such that there is a satisfiable schema of power \( n \) and every disjunction of two inequivalent schemata of power \( n \) has the same power?

(c) (15 points) What is the maximum power and what is the minimum power that can be achieved by a conjunction of two inequivalent schemata of power 64?

4. (15 points) For the purposes of this problem, we restrict attention to monadic quantification schemata (abbreviated MQ-schemata) all of whose predicate letters are among \( F \) and \( G \), and to structures which interpret exactly these predicate letters. We employ the following terminology.

- If \( S \) and \( T \) are MQ-schemata we say that a structure \( A \) is a *counterexample* to the claim that \( S \) implies \( T \) if and only if \( A \models S \) and \( A \not\models T \).

Let \( S \) be the schema 

\[ (\forall x)(Fx \oplus Gx) \]

and let \( T \) be the schema 

\[ (\forall x)Fx \oplus (\forall x)Gx. \]

How many structures with universe of discourse \( \{1,2,3,4,5\} \) are counterexamples to the claim that \( S \) implies \( T \)?