1. (10 points) How long a list of pure monadic schemata involving only the predicate letters “F” and “G” can be constructed so that no two schemata on the list are equivalent, and every schema on the list implies $(\forall x)(Fx \oplus Gx)$? 8

2. (10 points) How long a list of pure monadic schemata involving only the predicate letters “F” and “G” can be constructed so that no two schemata on the list are equivalent and each schema on the list is satisfied by exactly 228 structures with universe of discourse $\{1,2,3,4\}$? 16

3. Let $S_1$ be the following schema.

$$(\exists x)\neg Lxx \land (\forall x)(\forall y)(Lxy \supset Lyx).$$

(a) (10 points) Specify a structure $A_1$ of size at least 4 which satisfies $S_1$, that is, $U^{A_1}$ has at least 4 members and $A_1 \models S_1$.

$U^{A_1} = \{1,2,3,4\}$

$L^{A_1} = \emptyset$

(b) (10 points) How many structures with universe of discourse $\{1,2,3,4\}$ satisfy $S_1$? 960

4. Let $S_2$ be the following schema.

$$(\forall x)(\exists y)Lxy \land (\forall x)(\exists y)\neg Lxy.$$  

(a) (10 points) Specify a structure $A_2$ of size at least 4 which satisfies $S_2$.

$U^{A_2} = \{1,2,3,4\}$

$L^{A_2} = \{\langle i, i \rangle \mid 1 \leq i \leq 4\}$

(b) (10 points) How many structures with universe of discourse $\{1,2,3,4\}$ satisfy $S_2$? 38,416
5. Let $S_3$ be the following schema.

$$(\forall x)(\exists y)(\forall z)(Rxz \equiv z = y) \land (\forall x)(\forall y)(Rxy \supset \neg Ryx) \land (\forall x)(\forall y)(\forall z)((Rxy \land Ryz) \supset Rzx).$$

(a) (10 points) Specify a structure $A_3$ of size at least 4 which satisfies $S_3$.

$U^{A_3} = \{1, 2, 3, 4, 5, 6\}$

$R^{A_3} = \{\langle 1, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 1 \rangle, \langle 4, 5 \rangle, \langle 5, 6 \rangle, \langle 6, 4 \rangle\}$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5, 6\}$ satisfy $S_3$?

40

6. We say that a schema $S$ admits a positive natural number $n$ if and only if there is a structure $A$ of size $n$ which satisfies $S$.

(a) (10 points) Write down a schema $S$ involving only the dyadic predicate letter “$L$” and the identity predicate such that $S$ admits $n$ if and only if $n$ is divisible by two, and $S$ implies

$$(\forall x)Lxx \land (\forall x)(\forall y)(Lxy \supset Lyx) \land (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz)).$$

Let $S$ be the conjunction of the following two schemata.

- $(\forall x)Lxx \land (\forall x)(\forall y)(Lxy \supset Lyx) \land (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz))$
- $(\forall x)(\exists y)(\forall z)((Lxz \land x \neq z) \equiv z = y)$

(b) (10 points) Write down a schema $S$ involving only the dyadic predicate letter “$R$” and the identity predicate such that $S$ admits $n$ if and only if $n$ is divisible by three, and $S$ implies

$$(\forall x)(\exists y)(\forall z)(Rxz \equiv z = y).$$

Let $S$ be the conjunction of the following two schemata.

- $(\forall x)(\exists y)(\forall z)(Rxz \equiv z = y)$
- $(\forall x)(\forall y)(Rxy \supset \neg Ryx) \land (\forall x)(\forall y)(\forall z)((Rxy \land Ryz) \supset Rzx)$