1. (30 points) Taking the universe of discourse to be the set of all integers \{\ldots, -2, -1, 0, 1, 2, \ldots\} and using the triadic predicate letter “P” to express the relation \[3\] is the product of \[1\] and \[2\], and the dyadic predicate “L” to express the relation \[1\] is strictly less than \[2\], express the following statements in quantificational notation. (The boxed numerals indicate the order of argument places to the predicate letters.) You may need to use the symbol for identity in your paraphrases.

(a) \( x = |y| \).

(b) \( x \) is a negative integer.

(c) \( x = 8 \).

2. (40 points) Let \( S \) be the following schema.

\((\forall x)(\forall y)(L_{xy} \supset L_{yx}) \land (\forall x)\neg L_{xx} \land (\forall x)(\exists y)(\exists z)(y \neq z \land (\forall w)(L_{xw} \equiv (w = y \lor w = z))))\)

(a) How long a list of distinct structures \( A \) with universe of discourse \{1, 2, 3, 4, 5, 6, 7, 8\} satisfy the schema \( S \)?

(b) How long a list of pairwise non-isomorphic structures with universe of discourse \{1, 2, 3, 4, 5, 6, 7, 8\} satisfy the schema \( S \)?

(c) How long a list of distinct structures \( A \) with universe of discourse \{1, 2, 3, 4, 5, 6, 7, 8\} satisfy the condition: \( A \models S \) and \( |\text{Aut}(A)| = 64 \)?

(d) How long a list of distinct structures \( A \) with universe of discourse \{1, 2, 3, 4, 5, 6, 7, 8\} satisfy the condition: \( A \models S \) and \( |\text{Def}(A)| = 4 \)?
3. (30 points) For each of the following pairs consisting of a set of schemata $X$ and a schema $S$ determine whether $X$ implies $S$. If so, provide a deduction to establish the implication. If not, specify a structure which makes $S$ false and all the schemata in $X$ true.

(a) $X : \{(\forall x)(\forall y)(Lxy \supset Lyx), (\forall x)\neg Lxx, (\forall x)(\exists y)(\exists z)(y \neq z \land (\forall w)(Lxw \equiv (w = y \lor w = z))), (\forall x)(\exists y)(\forall z)(Fxz \equiv z = y), (\forall x)(\forall y)(\forall z)((Fxz \land Fyz) \supset x = y)\}$

$S : (\forall x)(\exists y)Fyx$

$A : U^A =
F^A =
L^A =$

Deduction
(b) $X: \{ (\forall x)(\exists y)Lxy, (\forall x)(\exists y)\neg Lxy, (\forall x)(\forall y)x = y \}$

$S: p \land \neg p$

$B: U^B =$

$L^B =$

Deduction
(c) \[ X : \{ (\forall x)(\forall y)(\text{Ly} \supset \text{Lx}), (\forall x)\neg \text{Lxx}, (\forall x)(\exists z)(\exists u)(\exists v)(y \neq z \land y \neq u \land y \neq v \land u \neq z \land v \neq z \land u \neq v \land (\forall w)(\text{Lxw} \equiv (w = y \lor w = z \lor w = u \lor w = v)), (\forall x)(\forall y)(\exists z)(\text{Lx} \land \text{Lyz}), (\forall x)(\forall y)(\exists z)(\neg \text{Lxz} \land \neg \text{Ly} \land z \neq y \land z \neq x), (\forall x)(\forall y)(x \neq y \supset (\exists z)(\text{Lxz} \land \neg \text{Lyz} \land z \neq y) \} \]

\[ S : p \land \neg p \]

\[ C : U^C = \]

\[ L^C = \]

Deduction