1 Lecture 11.09.08

Logic is the science of truth. Truth arises from relations between language and the world. Logic provides mathematical models of such relations. In logic, we study formal languages $L$ and structures $A$ which play the role of language and the world. The central relation is truth of a sentence $\varphi \in L$ in a structure $A$ (we write this as $A \models \varphi$, and we say $\varphi$ is true in $A$, or $A$ satisfies $\varphi$). In terms of this relation, we define the notion of logical consequence: if $\Sigma \subseteq L$ is a set of sentences and $\varphi \in L$ is a sentence we say $\varphi$ is a logical consequence of $\Sigma$ (and write $\Sigma \models \varphi$), if and only if,

$$\text{for all structures } A, \text{ if } A \models \Sigma, \text{ then } A \models \varphi.$$   

We can also define the set $V$ of valid sentences of $L$, namely, $\varphi \in L$ is valid, if and only if, for every structure $A$, $A \models \varphi$.

One important example of a formal language is first order logic. This language suffices for the formalization of large tracts of scientific discourse. A part of this course will be devoted to answering two interesting epistemological (even technological) questions concerning first order validity.

1. Can we find out that a sentence of first order logic is valid, if in fact it is?
2. Can we determine whether or not a sentence of first order logic is valid?

The answers to both these questions emerged from work of Kurt Gödel, Alonzo Church, and Alan Turing.

1. Gödel's Completeness Theorem: $V$ is semi-decidable.
2. Church-Turing Theorem: $V$ is not decidable.

Reading: Enderton, Sections 1.1 & 1.2
Exercises: Enderton, Exercise 1.2.10 (Please devote some attention to this exercise, since I would like to go over it during our next class meeting.)
For those who would like to proceed ahead, our next reading will be sections 1.5 and 1.7; exercises 1.7.1-1.7.3.