Isabelle Implementation of Protocol Composition Logic

Dan Auerbach    Cary Kempston
Anupam Datta    Ante Derek
John C. Mitchell

Stanford University
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Analysis of Protocols in PCL

1. Formalize the protocol: (Challenge-Response protocol)
   \[
   [\text{Init}_{\text{CR}}]_X = [\text{new } x; \text{ send } \hat{X}, \hat{Y}, x; \text{ receive } \hat{Y}, \hat{X}, y, z; \text{ match } z/\text{SIG}_{\hat{Y}}(y, x, \hat{X})]_{\hat{X}}
   \]

2. State the Security Property: (Authentication)
   \[
   \phi[\text{Init}_{\text{CR}}]_X \text{ Honest}(\hat{Y}) \land (\hat{X} \neq \hat{Y}) \supset \exists Y. \text{ActionsInOrder}(\text{Send}(X, \hat{X}, \hat{Y}, x), \text{Receive}(Y, \hat{X}, \hat{Y}, x), \text{Send}(Y, \hat{Y}, \hat{X}, y, \text{SIG}_{\hat{Y}}(y, x, \hat{X})), \text{Receive}(X, \hat{Y}, \hat{X}, y, \text{SIG}_{\hat{Y}}(y, x, \hat{X})))
   \]

3. Axiomatically prove security properties.

Applied to practical protocols: GDOI [Meadows-Pavlovic04], IKEv2, SSL/TLS, 802.11i (WIP)
PCL Syntax

\[
\begin{align*}
\text{name} & \quad ::= \quad \hat{X} \\
\text{thread} & \quad ::= \quad P \\
\text{term} & \quad ::= \quad x | \text{name} | ENC[K](t) | SIG[K](t) \\
\text{action} & \quad ::= \quad \epsilon | \text{send } t | \text{receive } t | \text{new } t | \text{match } t/t \\
\text{strand} & \quad ::= \quad \text{strand} ; \text{action} | \text{action} \\
\text{cord} & \quad ::= \quad \text{thread}[\text{strand}] \\
\text{formula} & \quad ::= \quad \text{Send}(P, t) | \text{Receive}(P, t) | \text{New}(P, t) | \text{Verify}(P, t) | \\
& \quad \text{Decrypt}(P, t) | \text{Has}(P, t) | \text{Fresh}(P, t) | \text{Honest}(N) | \\
& \quad \text{FirstSend}(P, t, t') | \text{Contains}(t_1, t_2) | \text{Start}(P) \\
\text{modal} & \quad ::= \quad \{ \text{formula, cord, formula} \}
\end{align*}
\]
PCL Axioms and Rules

AA1S \( \phi[\text{send } t]_X \text{ Send}(X, t) \)

ORIG \( \text{New}(X, x) \supset \text{Has}(X, x) \)

P1N \( \text{New}(X, t)[a]_X \text{ New}(X, t) \)

DEC \( \text{Has}(X, \text{ENC}[K](x)) \land \text{Has}(X, K) \supset \text{Has}(X, x) \)

SEC \( \text{Honest}(\hat{X}) \land \text{Decrypt}(Y, \text{ENC}[\hat{X}](x)) \supset (\hat{Y} = \hat{X}) \)

\[
\frac{\theta[P]_X \phi \quad \theta[P]_X \psi}{\theta[P]_X \phi \land \psi} \quad \frac{\theta[P]_X \phi \quad \theta' \supset \theta \quad \phi \supset \phi'}{\theta'[P]_X \phi'} \quad \frac{\phi_1[P]_A \phi_2 \quad \phi_2[P']_A \phi_3}{\phi_1[PP']_A \phi_3}
\]

G1 \(\text{G3} \quad \text{SEQ} \)
Isabelle

- Isabelle is a generic theorem-prover and logical framework [Paulson, 1989]
- Allows the implementation of new logics by specifying syntax and axioms.
- Better than implementing each logic from scratch; no need to supply methods for variable binding, rule instantiation, and proof.
- Structured proofs are available using interface packages Isar and ProofGeneral.
Encoding PCL in Isabelle

• The syntax and axioms are represented in a theory file:

```isabelle
consts
  PSend :: "[thread, CTerm] => o"
syntax
  PSend :: "[threadI, CTermList] => actformI"
    ("Send'(_,_')")
axioms
  AA1S: "\{ P, X[send t], Send(X,t) \}"
  REC : "Receive(X,t) --> Has(X,t)"
  SEQ: "\{ \{ P, X[S1], Q \}; \{ Q, X[S2], R \}\} \]
    ==> \{ P, X[S1 ; S2], R \}"
```
Sample Proof (1)

lemma "{P, X[new t; send t], Has(X,t) & Send(X,t)}";
proof -;
have A: "{P, X[new t; send t], Has(X,t)}";
  apply (rule G3);
  apply (rule SEQ);
  apply (rule AA1N);
  apply (rule P1N);
  apply (blast);
  apply (rule ORIG);
done;
Sample Proof (2)

have B: "\{P, X[new t; send t], Send(X,t)\}"
proof -;
  have C: "\{P, X[new t], New(X,t)\}" by (rule AA1N);
  have D: "\{New(X,t), X[send t], Send(X,t)\}"
    by (rule AA1S);
  from C D show "\{P, X[new t; send t], Send(X,t)\}"
    by (rule SEQ);
  qed;
  from A B show "\{P, X[new t; send t],
    Has(X,t) & Send(X,t)\}"
    by (rule G1);
  qed;
Current Status and Future Directions

• **Implementation:**
  – 322 line theory file: 200 lines of grammar, 122 lines of axioms and rules.
  – 45 axioms, 10 inference rules of PCL.
  – 10 auxiliary axioms (needed to work with Isabelle).

• **Future Directions:**
  – Automate(?) first order reasoning: interface with Isabelle’s automatic deduction capabilities.
  – Investigate decidability of PCL.
  – Distribute PCL implementation to other users.
  – Machine-check proofs of properties of practical protocols like GDOI, IKEv2, SSL, 802.11i.