LGIC 010 & PHIL 005
Problem Set 8
Spring Term, 2017
DUE IN CLASS MONDAY, APRIL 17

- $\mathbb{Z}$ is the set of integers $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
- Let A be a structure interpreting a triadic predicate letter P. h is an *automorphism* of A if and only if h is a bijection of U^A onto U^A and for all $i, j, k \in U^A$, $\langle i, j, k\rangle \in P^A$ if and only $\langle h(i), h(j), h(k)\rangle \in P^A$.
- Let A be a structure. $\mathsf{Aut}(A) = \{h \mid h \text{ is an automorphism of } A\}$.
- Let $S(x)$ be a schema with a single free variable x and A a structure.

$$S[A] = \{a \in U^A \mid A \models S[x|a]\}.$$

1. Let A be a structure interpreting a single triadic predicate letter P with $U^A = \mathbb{Z}$ and $P^A = \{\langle i, j, k\rangle \mid |i - j| = k\}$, the distance relation on $\mathbb{Z}$.
   (a) (25 points) Let $X_1 = \{i \in \mathbb{Z} \mid i < 0\}$. Is X_1 definable in A? If it is, write down a schema $S_1(x)$ such that $S_1[A] = X_1$; if not, specify an $h_1 \in \mathsf{Aut}(A)$ such that $h_1[X_1] \neq X_1$.
   (b) (25 points) Let $X_2 = \{1\}$. Is X_2 definable in A? If it is, write down a schema $S_2(x)$ such that $S_2[A] = X_2$; if not, specify an $h_2 \in \mathsf{Aut}(A)$ such that $h_2[X_2] \neq X_2$.
2. Let B be a structure interpreting a single triadic predicate letter Q with $U^B = \mathbb{Z}$ and $Q^B = \{\langle i, j, k\rangle \mid i + j = k\}$, the sum relation on $\mathbb{Z}$.
   (a) (25 points) Let $X_3 = \{i \in \mathbb{Z} \mid i < 0\}$. Is X_3 definable in B? If it is, write down a schema $S_3(x)$ such that $S_3[B] = X_3$; if not, specify an $h_3 \in \mathsf{Aut}(B)$ such that $h_3[X_3] \neq X_3$.
   (b) (25 points) Let $X_4 = \{i \in \mathbb{Z} \mid i \text{ is an odd integer}\}$. Is X_4 definable in B? If it is, write down a schema $S_4(x)$ such that $S_4[B] = X_4$; if not, specify an $h_4 \in \mathsf{Aut}(B)$ such that $h_4[X_4] \neq X_4$.