

LGIC 010 & PHIL 005

Problem Set 8

Spring Term, 2012

1. Let  $A$  be the structure interpreting a single dyadic predicate letter  $R$  with  $U^A = \{1, 2, 3\}$  and  $R^A = \{\langle 1, 2 \rangle, \langle 2, 2 \rangle, \langle 3, 2 \rangle\}$ .
  - (a) (10 points) List all the automorphisms of  $A$ .
  - (b) (10 points) List all sets which are definable in  $A$  along with schemata which define them.
2. (10 points) Let  $A$  be the structure interpreting a single dyadic predicate letter  $R$  with  $U^A = \{1, 2, 3\}$  and  $R^A = \{\langle 1, 2 \rangle, \langle 2, 2 \rangle, \langle 3, 2 \rangle\}$ . Write down a schema  $S$  so that for every structure  $B$ ,  $B$  satisfies  $S$  if and only if  $B$  is isomorphic to  $A$ .

3. Let  $S$  be the conjunction of the following schemata.

$$(\forall x)(\exists y)Rxy$$

$$(\forall x)(\forall y)(\forall z)((Rxy \wedge Rxz) \supset y = z)$$

- (a) (20 points) How long a list of pairwise non-isomorphic structures with universe of discourse  $\{1, 2, 3\}$  satisfy the schema  $S$ ?
- (b) (10 points) How long a list of structures with universe of discourse  $\{1, 2, 3\}$  satisfy the schema  $S$ ?
- (c) (20 points) Give an example of structures  $A$  and  $B$  such that
  - i.  $A$  and  $B$  both satisfy  $S$ ;
  - ii.  $A$  is not isomorphic to  $B$ ;
  - iii.  $U^A = U^B = \{1, 2, 3\}$ ;
  - iv. exactly four subsets of  $\{1, 2, 3\}$  are definable in  $A$  and exactly four subsets of  $\{1, 2, 3\}$  are definable in  $B$ .
- (d) (20 points) Give an example of a structure  $A$  such that
  - i.  $A$  satisfies  $S$ ;
  - ii.  $U^A = \{1, 2, 3\}$ ;
  - iii. exactly eight subsets of  $\{1, 2, 3\}$  are definable in  $A$ .