LGIC 010 & PHIL 005
Problem Set 8
Spring Term, 2011

1. Let A be the structure interpreting a single dyadic predicate letter R with $U^A = \{1, 2, 3\}$ and $R^A = \{\langle 1, 2\rangle, \langle 2, 1\rangle\}$.

   (a) (10 points) List all the automorphisms of A.

   (b) (10 points) List all sets which are definable in A along with schemata which define them.

2. (10 points) Let A be the structure interpreting a single dyadic predicate letter R with $U^A = \{1, 2, 3\}$ and $R^A = \{\langle 1, 2\rangle, \langle 2, 1\rangle\}$. Write down a schema S so that for every structure B, B satisfies S if and only if B is isomorphic to A.

3. Let S be the conjunction of the following schemata.

   $(\forall x)(\exists y)(\forall z)(Rxz \equiv y = z)$
   $(\forall x)(\forall y)(\forall z)((Rxz \wedge Ryz) \supset x = y)$

   (a) (20 points) How long a list of pairwise non-isomorphic structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy the schema S?

   (b) (10 points) How long a list of structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy the schema S?

   (c) (20 points) Give an example of structures A and B such that

      i. A and B both satisfy S;
      ii. A is not isomorphic to B;
      iii. $U^A = U^B = \{1, 2, 3, 4\}$;
      iv. exactly four subsets of $\{1, 2, 3, 4\}$ are definable in A and exactly four subsets of $\{1, 2, 3, 4\}$ are definable in B.

   (d) (20 points) Give an example of a structures A and B such that

      i. A and B both satisfy S;
      ii. $U^A = U^B = \{1, 2, 3, 4\}$;
      iii. exactly six structures with universe of discourse $\{1, 2, 3, 4\}$ are isomorphic to A and exactly six structures with universe of discourse $\{1, 2, 3, 4\}$ are isomorphic to B;
      iv. the number of subsets of $\{1, 2, 3, 4\}$ that are definable in A is not equal to the number of subsets of $\{1, 2, 3, 4\}$ that are definable in B.