LGIC 010 & PHIL 005
Problem Set 7
Spring Term, 2018
DUE IN CLASS MONDAY, APRIL 9

Definitions:

- $[n] = \{1, \ldots, n\}$.
- $\mathsf{mod}(S, n) = \{A \mid A \models S \text{ and } U^A = [n]\}$.
- $A \cong B$ if and only if A is *isomorphic to* B.
- A list l of structures is *succinct* if and only if for every pair of distinct structures A and B appearing on l, $A \ncong B$.
- $\mathsf{Aut}(A) = \{h \mid h \text{ is an automorphism of } A\}$.
- $\mathsf{orb}(a, A) = \{h(a) \mid h \in \mathsf{Aut}(A)\}$.
- $\mathsf{Orbs}(A) = \{O \mid \text{for some } a \in U^A,\ O = \mathsf{orb}(a, A)\}$.
- Let $S(x)$ be a schema with a single free variable x and A a structure.

$$S[A] = \{a \in U^A \mid A \models S[x|a]\}.$$

In part (a) of the following problems, for each structure A on your list, please specify the extension L^A explicitly as a set of ordered pairs. You may also draw an "arrow diagram" of A to aid the reader.

1. Let S_1 be the conjunction of the following schema.
   - $(\forall x)(\exists y)Lxy$
   - $(\forall x)(\forall y)(\forall z)((Lxy \wedge Lxz) \supset y = z)$
   - $(\forall x)(\forall y)(\forall z)((Lyx \wedge Lzx) \supset y = z)$

   (a) (25 points) Construct a maximal length succinct list l_1 of structures such that each structure listed on l_1 is a member of $\mathsf{mod}(S_1, 4)$.

   (b) (25 points) For each structure A on your list l_1 and each $O \in \mathsf{Orbs}(A)$ write down a schema $S(x)$ such that $S[A] = O$.

2. Let S_2 be the conjunction of the following schemata.
   - $(\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)\neg Lxx$
   - $(\forall x)(\exists y)(\exists z)(y \neq z \wedge (\forall w)(Lxw \equiv (w = y \vee w = z)))$

   (a) (25 points) Construct a maximal length succinct list l_2 of structures such that each structure listed on l_2 is a member of $\mathsf{mod}(S_2, 8)$.

   (b) (25 points) For each structure A on your list l_2 and each $O \in \mathsf{Orbs}(A)$ write down a schema $S(x)$ such that $S[A] = O$.