

**LGIC 010 & PHIL 005**  
**Problem Set 7**  
**Spring Term, 2017**  
**DUE IN CLASS MONDAY, APRIL 10**

Definitions:

- $[n] = \{1, \dots, n\}$ .
- $\text{mod}(S, n) = \{A \mid A \models S \text{ and } U^A = [n]\}$ .
- $A \cong B$  if and only if  $A$  is *isomorphic* to  $B$ .
- A list  $l$  of structures is *succinct* if and only if for every pair of distinct structures  $A$  and  $B$  appearing on  $l$ ,  $A \not\cong B$ .
- $\text{Aut}(A) = \{h \mid h \text{ is an automorphism of } A\}$ .
- $\text{orb}(a, A) = \{h(a) \mid h \in \text{Aut}(A)\}$ .
- $\text{Orbs}(A) = \{O \mid \text{for some } a \in U^A, O = \text{orb}(a, A)\}$ .
- Let  $S(x)$  be a schema with a single free variable  $x$  and  $A$  a structure.

$$S[A] = \{a \in U^A \mid A \models S[x|a]\}.$$

1. Let  $S_1$  be the following schema.

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv z = y)$$

- (a) (25 points) Construct a maximal length succinct list  $l_1$  of structures such that each structure listed on  $l_1$  is a member of  $\text{mod}(S_1, 3)$ .
- (b) (25 points) For each structure  $A$  on your list  $l_1$  and each  $O \in \text{Orbs}(A)$  write down a schema  $S(x)$  such that  $S[A] = O$ .

2. Let  $S_2$  be the conjunction of the following schemata.

- $(\forall x)(\forall y)(Lxy \supset \neg Lyx)$
- $(\forall x)(\forall y)(Lxy \supset (Lyz \supset Lxz))$

- (a) (25 points) Construct a maximal length succinct list  $l_2$  of structures such that each structure listed on  $l_2$  is a member of  $\text{mod}(S_2, 3)$ .
- (b) (25 points) For each structure  $A$  on your list  $l_2$  and each  $O \in \text{Orbs}(A)$  write down a schema  $S(x)$  such that  $S[A] = O$ .