

**LGIC 010 & PHIL 005**  
**Problem Set 6**  
**Spring Term, 2018**  
**DUE IN CLASS MONDAY, MARCH 19**

We write  $\mathbb{Z}^+$  for the set of positive integers  $\{1, 2, 3, \dots\}$ . The *spectrum* of a schema  $S$  (written  $\text{Spec}(S)$ ) is defined as follows.

$$\text{Spec}(S) = \{n \in \mathbb{Z}^+ \mid \text{mod}(S, n) \neq \emptyset\}.$$

1. (25 points) Write down a schema  $S_1$  involving only the dyadic predicate letter “ $L$ ,” and the identity predicate such that

- $S_1$  implies  $(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx)$  and
- $\text{Spec}(S_1) = \{n \in \mathbb{Z}^+ \mid \text{for some } i \in \mathbb{Z}^+, n = 4i\}$ .

2. (25 points) Write down a schema  $S_2$  involving only the dyadic predicate letter “ $L$ ” and the identity predicate such that

- $S_2$  implies  $(\forall x)(\exists y)(\forall z)(Lxz \equiv z = y) \wedge (\forall x)(\forall y)((Lxy \wedge Lyx) \supset x = y)$  and
- $\text{Spec}(S_2) = \{n \in \mathbb{Z}^+ \mid n \text{ is even}\}$ .

3. (25 points) Let  $S_3$  be the conjunction of the following schemata.

- $(\forall x)(\forall y)(Pxy \supset \neg Pyx)$
- $(\forall x)(\forall y)(\forall z)((Pxy \wedge Pyz) \supset Pxz)$
- $(\forall x)(\forall y)(\forall z)((Pxz \wedge Pyz \wedge x \neq y) \supset (Pxy \vee Pyx))$
- $(\exists x)(\forall y)((\forall z)\neg Pzy \equiv y = x)$
- $(\forall x)(\forall y)(Lxy \equiv (Pxy \wedge (\forall z)(\neg(Pxz \wedge Pzy))))$
- $(\forall x)((\forall y)\neg Pxy \vee (\exists y)(\exists z)(y \neq z \wedge (\forall u)(Lxu \equiv (u = y \vee u = z))))$
- $(\forall x)Exx$
- $(\forall x)(\forall y)(Exy \supset Eyx)$
- $(\forall x)(\forall y)(\forall z)(Exy \supset (Eyz \supset Exz))$
- $(\forall x)(\forall y)(x \neq y \supset (Exy \equiv ((\exists z)(\exists w)(Ezw \wedge Lzx \wedge Lwy))))$
- $(\forall x)(\forall y)(Exy \supset ((\exists z)Lxz \equiv (\exists z)Lyz))$

Specify the spectrum of  $S_3$ .

$\text{Spec}(S_3) =$

4. (25 points) Let  $S_4$  be the conjunction of the following schemata.

- $(\forall x)(\forall y)(\forall z)(Hxyz \supset (Fx \wedge Fy))$
- $(\forall x)(\forall y)((Fx \wedge Fy) \supset (\exists z)(\forall w)(Hxyw \equiv w = z))$
- $(\forall z)(\exists x)(\exists y)Hxyz$
- $(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Hvwz \wedge Hxyz) \supset (v = x \wedge w = y))$

Specify the spectrum of  $S_4$ .

$\text{Spec}(S_4) =$