LGIC 010 & PHIL 005
Problem Set 6
Second Corrected Version
Spring Term, 2017
DUE IN CLASS MONDAY, MARCH 20

We write $\mathbb{Z}^+$ for the set of positive integers $\{1, 2, 3, \ldots\}$. The *spectrum* of a schema S (written $\mathsf{Spec}(S)$) is defined as follows.

$$\mathsf{Spec}(S) = \{n \in \mathbb{Z}^+ \mid \mathsf{mod}(S, n) \neq \emptyset\}.$$

1. (25 points) Write down a schema S involving only the dyadic predicate letter "L," and the identity predicate such that
   - S implies $(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset \neg Lyx)$ and
   - $\mathsf{Spec}(S) = \{n \in \mathbb{Z}^+ \mid \text{ for some } i \in \mathbb{Z}^+,\ n = 4i\}$.

2. (25 points) Recall that SLO is the conjunction of the following schemata.
   - $(\forall x)(\forall y)(Lxy \supset \neg Lyx)$
   - $(\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz))$
   - $(\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx))$

   Write down a schema S involving only the dyadic predicate letter "L," the monadic predicate letter "F," and the identity predicate such that
   - S implies SLO and
   - $\mathsf{Spec}(S) = \{n \in \mathbb{Z}^+ \mid \text{ for some } i \in \mathbb{Z}^+,\ n = 3i\}$.

3. (25 points) Let S_1 be the conjunction of the following schemata.

   - $(\forall x)(\forall y)(Lxy \supset (Fx \wedge \neg Fy))$
   - $(\forall x)(Fx \supset (\exists y)(\forall w)(Lxw \equiv w = y))$
   - $(\forall x)(\neg Fx \supset (\exists y)(\exists z)(y \neq z \wedge (\forall w)(Lwx \equiv (w = y \vee w = z))))$

   Specify the spectrum of S_1.

   $\mathsf{Spec}(S_1) =$

4. (25 points) Let S_2 be the conjunction of the following schemata.

   - $(\exists x)Fx$
   - $(\forall x)(\forall y)(\forall z)(Hxyz \supset (Fy \wedge Fz))$
   - $(\forall x)(\forall y)\neg Hxyy$
   - $(\forall x)(\forall y)(\forall z)(\forall w)((Hxyz \wedge Hxzw) \supset Hxyw)$
   - $(\forall x)(\forall y)(\forall z)((Fy \wedge Fz \wedge y \neq z) \supset (Hxyz \vee Hxzy))$
   - $(\forall x)(\forall y)((\forall z)(\forall w)(Hxzw \equiv Hyzw) \supset x = y)$
   - $(\forall x)(\forall y)(\forall z)(Hxyz \supset (\exists w)(Hwzy \wedge (\forall u)(\forall v)((u \neq y \wedge u \neq z \wedge v \neq y \wedge v \neq z) \supset (Hxuv \equiv Hwuv))))$

   Specify the spectrum of S_2.

   $\mathsf{Spec}(S_2) =$