**LGIC 010 & PHIL 005**
**Problem Set 6**
**Spring Term, 2014**

We say that a schema $S$ *admits* a positive natural number $n$ if and only if there is a structure $A$ of size $n$ which satisfies $S$.

1. (25 points) Write down a schema $S$ involving only the dyadic predicate letter "$R$" and the identity predicate such that $S$ admits $n$ if and only if $n$ is divisible by three, and $S$ implies
$$(\forall x)\neg Rxx \wedge (\forall x)(\forall y)(Rxy \supset Ryx).$$

2. (25 points) Write down a schema $S$ involving only the dyadic predicate letter "$R$," the monadic predicate letter "$F$," and the identity predicate such that $S$ admits $n$ if and only if $n$ is even, and $S$ implies
$$(\forall x)\neg Rxx \wedge (\forall x)(\forall y)(\forall z)(Rxy \supset (Ryz \supset Rxz)) \wedge (\forall x)(\forall y)(x \neq y \supset (Rxy \vee Ryx)).$$

3. (25 points) Write down a schema $S$ involving only the monadic predicate letters "$F$" and "$G$," the triadic predicate letter "$H$," and the identity predicate such that $S$ admits $n$ if and only if $n$ is a composite, that is, if and only if $n = i \cdot j$, for some $1 < i, j < n$, and $S$ implies

$$(\forall x)(\forall y)(\forall z)(Hxyz \supset (Fx \wedge Gy)) \wedge (\forall x)(\forall y)((Fx \wedge Gy) \supset (\exists z)(\forall w)(Hxyw \equiv w = z)).$$

4. (25 points) Write down a schema $S$ involving only the dyadic predicate letter "$R$" and the identity predicate such that $S$ admits $n$ if and only if $n$ is odd, and $S$ implies

$$(\forall x)(\exists y)(\forall z)(Rxz \equiv z = y).$$