LGIC 010 & PHIL 005
Problem Set 6
Spring Term, 2013

We say that a schema S *admits* a positive natural number n if and only if there is a structure A of size n which satisfies S.

1. (25 points) Write down a schema S involving only the dyadic predicate letter "R," and the identity predicate such that S admits n if and only if n is even, and S implies

$$(\forall x)\neg Rxx \wedge (\forall x)(\forall y)(Rxy \supset Ryx).$$

2. (25 points) Write down a schema S involving only the dyadic predicate letter "R," and the identity predicate such that S admits n if and only if n is odd, and S implies

$$(\forall x)\neg Rxx \wedge (\forall x)(\forall y)(Rxy \supset Ryx).$$

3. (25 points) Write down a schema S involving only the monadic predicate letters "F" and "G," the triadic predicate letter "H," and the identity predicate such that S admits n if and only if n is a positive power of 2, that is, if and only if $n = 2^i$, for some $i \geq 1$, and S implies

$$(\forall x)(\forall y)(\forall z)(Hxyz \supset (Fy \wedge Gz)) \wedge (\forall x)(\forall y)(Fy \supset (\exists z)(\forall w)(Hxyw \equiv w = z)).$$

4. (25 points) Write down a schema S involving only the dyadic predicate letter "R," and the identity predicate such that S admits n if and only if n is divisible by three, and S implies

$$(\forall x)(\exists y)(\forall z)(Rxz \equiv z = y).$$