LGIC 010 & PHIL 005
Problem Set 6
Spring Term, 2012

We say that a schema S *admits* a positive natural number n if and only if there is a structure A of size n which satisfies S.

1. (25 points) Write down a schema S involving only the monadic predicate letter "F," the dyadic predicate letter "R," and the identity predicate such that S admits n if and only if n is odd, and S implies

$$(\forall x)\neg Rxx \wedge (\forall x)(\forall y)(\forall z)((Rxy \wedge Ryz) \supset Rxz) \wedge (\forall x)(\forall y)(x \neq y \supset (Rxy \vee Ryx)).$$

2. (25 points) Write down a schema S involving only the dyadic predicate letter "R" and the identity predicate such that S admits n if and only if n is divisible by three, and S implies

$$(\forall x)(\exists y)(\forall z)(Rxz \equiv y = z) \wedge (\forall x)(\exists y)Ryx.$$

3. (25 points) Recall that a positive natural number n is a perfect square if and only if for some m, $n = m^2$. Write down a schema S involving only the monadic predicate letter "F," the triadic predicate letter "H," and the identity predicate such that S admits n if and only if n is a perfect square, and S implies

$$(\forall x)(\forall y)(\forall z)(Hxyz \supset (Fx \wedge Fy)) \wedge (\forall x)(\forall y)((Fx \wedge Fy) \supset (\exists z)(\forall w)(Hxyw \equiv w = z)).$$

4. (25 points) Write down a schema S involving only the dyadic predicate letter "R" and the identity predicate such that S admits n if and only if n is even, and S implies

$$(\forall x)Rxx \wedge (\forall x)(\forall y)(\forall z)(Rxy \supset (Ryz \supset Rxz)) \wedge (\forall x)(\forall y)(Rxy \supset Ryx).$$