

LGIC 010 & PHIL 005

Problem Set 6

Spring Term, 2009

We say that a schema  $S$  admits a positive natural number  $n$  if and only if there is a structure  $A$  of size  $n$  which satisfies  $S$ .

1. (25 points) Write down a schema  $S$  involving only the dyadic predicate letter “ $R$ ,” and the identity predicate such that  $S$  admits  $n$  if and only if  $n$  is divisible by four, and  $S$  implies

$$(\forall x)\neg Rxx \wedge (\forall x)(\forall y)(Rxy \supset Ryx).$$

2. (25 points) Write down a schema  $S$  involving only the dyadic predicate letter “ $R$ ,” and the identity predicate such that  $S$  admits  $n$  if and only if  $n$  is odd, and  $S$  implies

$$(\forall x)(\exists y)(\forall z)(Rxz \equiv y = z).$$

3. (25 points) Recall that a positive natural number  $n$  is composite if and only if  $n$  is divisible by  $m$  for some  $1 < m < n$ . Write down a schema  $S$  involving only the monadic predicate letters “ $F$ ” and “ $G$ ,” the triadic predicate letter “ $H$ ,” and the identity predicate such that  $S$  admits  $n$  if and only if  $n$  is composite, and  $S$  implies

$$(\forall x)(\forall y)(\forall z)(Hxyz \supset (Fx \wedge Gy)) \wedge (\forall x)(\forall y)((Fx \wedge Gy) \supset (\exists z)(\forall w)(Hxyw \equiv w = z)).$$

4. (25 points) Write down a schema  $S$  involving only the dyadic predicate letter “ $R$ ” and the identity predicate such that  $S$  admits  $n$  if and only if  $n$  is even, and  $S$  implies

$$(\forall x)Rxx \wedge (\forall x)(\forall y)(\forall z)(Rxy \supset (Ryz \supset Rxz)) \wedge (\forall x)(\forall y)(Rxy \supset Ryx).$$