LGIC 010 & PHIL 005
Problem Set 5
Spring Term, 2016
DUE IN CLASS MONDAY, MARCH 14

We deploy the following concepts in formulating some of the problems below.

- If X is a finite set, we write $|X|$ for the number of members of X.
- A *graph* is a structure interpreting one dyadic predicate letter L.
- We use
  - Irr to abbreviate the schema $(\forall x)\neg Lxx$,
  - Sym to abbreviate the schema $(\forall x)(\forall y)(Lxy \supset Lyx)$, and
  - SG to abbreviate the conjunction of Irr and Sym. Structures that satisfy SG are called *simple graphs*.
- The *order* of a graph A (written $\mathsf{o}rd(A)$) is $|U^A|$. The *size* of a simple graph A (written $\mathsf{s}ize(A)$) is $|L^A|/2$. This corresponds to the number of "undirected edges" of A.
- If S is a schema, we write $\mathsf{mod}(S, n)$ for the set of structures A such that $A \models S$ and $U^A = \{1, \dots, n\}$.
- Let K be a set of simple graphs. We call A a *size maximal* member of K if and only if $A \in K$ and for every $B \in K$, $\mathsf{s}ize(A) \geq \mathsf{s}ize(B)$.
- For $n \geq 2$, we let $\Delta_n(x_1, \dots, x_n)$ abbreviate the schema:

$$x_1 \neq x_2 \wedge x_1 \neq x_3 \dots \wedge x_{n-1} \neq x_n.$$

- For $n \geq 3$, we let C_n abbreviate the schema:

$$(\exists x_1) \dots (\exists x_n)(\Delta_n(x_1, \dots, x_n) \wedge Lx_1x_2 \wedge Lx_2x_3 \wedge \dots L_{n-1}x_n \wedge Lx_nx_1).$$

1. Let S_1 be $\mathsf{SG} \wedge \neg C_3 \wedge \neg C_4 \wedge \neg C_5 \wedge \neg C_6$.

   (a) (10 points) Specify a structure A_1 which is a size maximal member of $\mathsf{mod}(S_1, 6)$.

   $U^{A_1} =$

   $L^{A_1} =$

(b) (10 points) How many structures are size maximal members of $\mathsf{mod}(S_1, 6)$?

2. Let S_2 be the conjunction of SG and $\neg C_3$.

(a) (10 points) Specify a structure A_2 which is a size maximal member of $\mathsf{mod}(S_2, 6)$.

$U^{A_2} =$

$L^{A_2} =$

(b) (10 points) How many structures are size maximal members of $\mathsf{mod}(S_2, 6)$?

3. Let S_3 be the following schema.

$(\forall x)(\forall y)(Lxy \supset \neg Lyx) \wedge (\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx)) \wedge \neg(\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz)$

(a) (10 points) Specify a structure $A_3 \in \mathsf{mod}(S_3, 4)$.

$U^{A_3} =$

$L^{A_3} =$

(b) (10 points) What is the value of $|\mathsf{mod}(S_3, 4)|$?

4. Let S_4 be the conjunction of the following four schemata.

- $(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Rvwz \wedge Rxyz) \supset (v = x \wedge w = y))$
- $(\forall x)(\forall y)(\forall z)(Rxyz \supset (Fx \wedge Fy))$
- $(\forall x)(\forall y)((Fx \wedge Fy) \supset (\exists z)(\forall w)(Rxyw \equiv w = z))$
- $(\forall z)(\exists x)(\exists y)Rxyz$

(a) (5 points) For which values of n strictly between 1 and 10 is $\mathsf{mod}(S_4, n)$ nonempty?

(b) (5 points) For one such n, specify a structure $A_4 \in \mathsf{mod}(S_4, n)$.

$U^{A_4} =$

$F^{A_4} =$

$R^{A_4} =$

(c) (10 points) For your chosen value of n, what is the value of $|\mathsf{mod}(S_4, n)|$?

5. Let S_5 be the conjunction of the schemata SG and $(\forall x)(\forall y)(\exists z)(Lxz \wedge Lyz)$.

(a) (10 points) What is the value of $|\mathsf{mod}(S_5, 4)|$?

(b) (10 points) Circle the number to which the ratio $|\mathsf{mod}(S_5, 40)|/2^{\binom{40}{2}}$ is nearest.

0 1/5 2/5 3/5 4/5 1