

**LGIC 010 & PHIL 005**  
**Problem Set 5**  
**Spring Term, 2012**

1. Let  $S_1$  be the following schema.

$$(\forall x)(\exists y)Lxy$$

(a) (10 points) Specify a structure  $A_1$  of size at least 3 which satisfies  $S_1$ , that is,  $U^{A_1}$  has at least 3 members and  $A_1 \models S_1$ .

$$U^{A_1} =$$

$$L^{A_1} =$$

(b) (10 points) How many structures with universe of discourse  $\{1, 2, 3\}$  satisfy  $S_1$ ?

2. Let  $S_2$  be the following schema.

$$(\forall x)(\exists y)Lxy \wedge (\forall y)(\exists x)Lxy$$

(a) (10 points) Specify a structure  $A_2$  of size at least 3 which satisfies  $S_2$ .

$$U^{A_2} =$$

$$L^{A_2} =$$

(b) (10 points) How many structures with universe of discourse  $\{1, 2, 3\}$  satisfy  $S_2$ ?

3. Let  $S_3$  be the following schema.

$$(\forall x)(\forall y)(Lxy \supset (Px \wedge \neg Py)) \wedge (\forall x)(Px \supset (\exists y)Lxy) \wedge (\forall x)(\neg Px \supset (\exists y)Lyx)$$

(a) (10 points) Specify a structure  $A_3$  of size at least 3 which satisfies  $S_3$ .

$$U^{A_3} =$$

$$L^{A_3} =$$

$$P^{A_3} =$$

(b) (10 points) How many structures with universe of discourse  $\{1, 2, 3\}$  satisfy  $S_3$ ?

4. Let  $S_4$  be the following schema.

$$(\forall x)Lxx \wedge (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz)) \wedge (\forall x)(\forall y)(Lxy \supset Lyx)$$

(a) (10 points) Specify a structure  $A_4$  of size at least 3 which satisfies  $S_4$ .

$$U^{A_4} =$$

$$L^{A_4} =$$

(b) (10 points) How many structures with universe of discourse  $\{1, 2, 3\}$  satisfy  $S_4$ ?

5. Let  $S_5$  be the conjunction of the following five schemata.

- $(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Rvwz \wedge Rxyz) \supset (v = x \wedge w = y))$
- $(\forall x)(\forall y)(\forall z)(Rxyz \supset (Fx \wedge Gy))$
- $(\forall x)(\forall y)((Fx \wedge Gy) \supset (\exists z)(\forall w)(Rxyw \equiv w = z))$
- $(\forall z)(\exists x)(\exists y)Rxyz$
- $(\exists x)(\exists y)(Fx \wedge Fy \wedge x \neq y) \wedge (\exists x)(\exists y)(Gx \wedge Gy \wedge x \neq y)$

(a) (10 points) Specify a structure  $A_5$  of size at least 3 which satisfies  $S_5$ .

$$U^{A_5} =$$

$$F^{A_5} =$$

$$G^{A_5} =$$

$$R^{A_5} =$$

(b) (10 points) How many structures with universe of discourse  $\{1, 2, 3\}$  satisfy  $S_5$ ?