**LGIC 010 & PHIL 005**
**Problem Set 5**
**Spring Term, 2011**

1. Let $S_1$ be the following schema.

$$(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)(\exists y)(\forall z)(Lxz \equiv y = z)$$

   (a) (10 points) Specify a structure $A_1$ of size at least 4 which satisfies $S_1$, that is, $U^{A_1}$ has at least 4 members and $A_1 \models S_1$.

   $U^{A_1} =$

   $L^{A_1} =$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy $S_1$?

2. Let $S_2$ be the following schema.

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z) \wedge (\forall y)(\exists x)Lxy$$

   (a) (10 points) Specify a structure $A_2$ of size at least 4 which satisfies $S_2$.

   $U^{A_2} =$

   $L^{A_2} =$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy $S_2$?

3. Let $S_3$ be the following schema.

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z) \wedge \neg(\forall y)(\exists x)Lxy \wedge (\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y)$$

(a) (10 points) Specify a structure $A_3$ of size at least 4 which satisfies $S_3$.

$U^{A_3} =$

$L^{A_3} =$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy $S_3$?

4. Let $S_4$ be the following schema.

$$(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz)) \wedge (\forall x)(\forall y)(Lxy \vee Lyx \vee x = y)$$

(a) (10 points) Specify a structure $A_4$ of size at least 4 which satisfies $S_4$.

$U^{A_4} =$

$L^{A_4} =$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy $S_4$?

5. Let $S_5$ be the conjunction of the following four schemata.

   - $(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Rvwz \wedge Rxyz) \supset (v = x \wedge w = y))$
   - $(\forall x)(\forall y)(\forall z)(Rxyz \supset (Fx \wedge Fy))$
   - $(\forall x)(\forall y)((Fx \wedge Fy) \supset (\exists z)(\forall w)(Rxyw \equiv w = z))$
   - $(\forall z)(\exists x)(\exists y)Rxyz$

   (a) (10 points) Specify a structure $A_5$ of size at least 4 which satisfies $S_5$.

   $U^{A_5} =$

   $F^{A_5} =$

   $R^{A_5} =$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy $S_5$?