

LGIC 010 & PHIL 005
Problem Set 5
Spring Term, 2009

1. Let S_1 be the following schema.

$$(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)(\exists y)(\forall z)(Lxz \equiv y = z)$$

(a) (10 points) Specify a structure A_1 of size at least 4 which satisfies S_1 , that is, U^{A_1} has at least 4 members and $A_1 \models S_1$.

$$U^{A_1} =$$

$$L^{A_1} =$$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_1 ?

2. Let S_2 be the following schema.

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z) \wedge (\forall y)(\exists x)Lxy$$

(a) (10 points) Specify a structure A_2 of size at least 4 which satisfies S_2 .

$$U^{A_2} =$$

$$L^{A_2} =$$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_2 ?

3. Let S_3 be the following schema.

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z) \wedge \neg(\forall y)(\exists x)Lxy \wedge (\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y)$$

(a) (10 points) Specify a structure A_3 of size at least 4 which satisfies S_3 .

$$U^{A_3} =$$

$$L^{A_3} =$$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_3 ?

4. Let S_4 be the following schema.

$$(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz)) \wedge (\forall x)(\forall y)(Lxy \vee Lyx \vee x = y)$$

(a) (10 points) Specify a structure A_4 of size at least 4 which satisfies S_4 .

$$U^{A_4} =$$

$$L^{A_4} =$$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_4 ?

5. Let S_5 be the conjunction of the following four schemata.

- $(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Rvwz \wedge Rxyz) \supset (v = x \wedge w = y))$
- $(\forall x)(\forall y)(\forall z)(Rxyz \supset (Fx \wedge Fy))$
- $(\forall x)(\forall y)((Fx \wedge Fy) \supset (\exists z)(\forall w)(Rxyw \equiv w = z))$
- $(\forall z)(\exists x)(\exists y)Rxyz$

(a) (10 points) Specify a structure A_5 of size at least 4 which satisfies S_5 .

$$U^{A_5} =$$

$$F^{A_5} =$$

$$R^{A_5} =$$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_5 ?