LGIC 010 & PHIL 005
Problem Set 4
Spring Term, 2019
DUE IN CLASS MONDAY, MARCH 11

For the purposes of this problem set, we restrict attention to pure monadic quantificational schemata all of whose predicate letters are among F and G, and to structures which interpret exactly these predicate letters. We employ the following terminology in the problems below.

- A pure monadi schema S implies a pure monadic schema T if and only if for every structure A, if $A \models S$, then $A \models T$. S and T are equivalent if and only if each implies the other.
- A list of pure monadic schemata is *succinct* if and only if no two schemata on the list are equivalent.
- A pure monadic schema *implies a list of schemata* if and only if it implies every schema on the list.
- The *power* of a pure monadic schema is the length of a longest succinct list of pure monadic schemata it implies.
- If X is a finite set, we write $|X|$ for the number of members of X.
- If S is a schema, we write $\mathsf{mod}(S, n)$ for the set of structures A such that $A \models S$ and $U^A = \{1, \ldots, n\}$.

1. (25 points) What is the length of a longest succinct list of schemata none of which imply the schema $(\forall x)(Fx \oplus Gx)$?

2. (25 points) What is the length of a longest succinct list of schemata all of which have power strictly less than the power of $(\exists x)(Fx \vee Gx)$?

3. (25 points) What is the length of a longest succinct list of schemata L, such that for every schema S on the list L, $|\mathsf{mod}(S, 4)| \leq 4$?

4. (25 points) What is the length of a longest succinct list of schemata L such that every schema S on the list L satisfies the following properties?
   - $|\mathsf{mod}(S, 4)| = 0$.
   - $|\mathsf{mod}(S, 5)| \neq 0$.