

**LGIC 010 & PHIL 005**  
**Problem Set 4**  
**Spring Term, 2018**  
**DUE IN CLASS MONDAY, FEBRUARY 19**

For the purposes of this problem set, we restrict attention to pure monadic quantificational schemata all of whose predicate letters are among  $F$  and  $G$ , and to structures which interpret exactly these predicate letters. We employ the following terminology in the problems below.

- A list of pure monadic schemata is *succinct* if and only if no two schemata on the list are equivalent.
  - A pure monadic schema *implies a list of schemata* if and only if it implies every schema on the list.
  - The *power* of a pure monadic schema is the length of a longest succinct list of pure monadic schemata it implies.
  - If  $X$  is a finite set, we write  $|X|$  for the number of members of  $X$ .
  - If  $S$  is a schema, we write  $\text{mod}(S, n)$  for the set of structures  $A$  such that  $A \models S$  and  $U^A = \{1, \dots, n\}$ .
1. (25 points) What is the length of a longest succinct list of schemata none of which imply the schema  $(\forall x)(Fx \wedge Gx)$ ?
  2. (25 points) What is the length of a longest succinct list of schemata all of which have power strictly less than the power of  $(\exists x)(Fx \wedge Gx)$ ?
  3. (25 points) What is the length of a longest succinct list of schemata  $S$  such that  $|\text{mod}(S, 4)| = 253$ ?
  4. (25 points) What is the length of a longest succinct list of schemata  $S$  all of which satisfy the following properties?
    - $|\text{mod}(S, 2)| = 0$ .
    - $|\text{mod}(S, 3)| \neq 0$ .