

LGIC 010 & PHIL 005
Problem Set 3
Spring Term, 2017
DUE IN CLASS MONDAY, FEBRUARY 6

For the purposes of this problem set, we restrict attention to monadic quantificational schemata (abbreviated MQ-schemata) all of whose predicate letters are among F , G and H , and to structures which interpret exactly these predicate letters. We employ the following terminology in the problems below.

- If S and T are MQ-schemata we say that a structure A is a *counterexample* to the claim that S implies T if and only if $A \models S$ and $A \not\models T$.
- If S and T are MQ-schemata we say that a structure A *witnesses the inequivalence of* S and T if and only if either A is a counterexample to the claim that S implies T or A is a counterexample to the claim that T implies S .

1. Let S be the schema

$$(\exists x)(Fx \wedge Gx) \wedge (\exists x)(Fx \wedge \neg Gx) \wedge (\exists x)(\neg Fx \wedge Hx)$$

and let T be the schema

$$(\forall x)((Fx \oplus Gx) \oplus Hx).$$

- (a) (25 points) How many structures with universe of discourse $\{1, 2\}$ are counterexamples to the claim that S implies T ?
 - (b) (25 points) How many structures with universe of discourse $\{1, 2, 3\}$ are counterexamples to the claim that S implies T ?
2. (25 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ witness the inequivalence of

$$(\forall x)((Fx \oplus Gx) \oplus Hx) \text{ and } (\forall x)((Fx \equiv Gx) \vee (Fx \equiv Hx) \vee (Gx \equiv Hx))?$$

3. (25 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ witness the inequivalence of $(\exists x)((Fx \oplus Gx) \oplus Hx)$ and $(\forall x)(Fx \vee \neg Fx)$?