LGIC 010 & PHIL 005
Problem Set 3
Spring Term, 2016
DUE IN CLASS MONDAY, FEBRUARY 8

For the purposes of this problem set, we restrict attention to monadic quantificational schemata (abbreviated MQ-schemata) all of whose predicate letters are among F and G, and to structures which interpret exactly these predicate letters. We employ the following terminology in the problems below.

- If S and T are MQ-schemata we say that a structure A is a *counterexample* to the claim that S implies T if and only if $A \models S$ and $A \not\models T$.
- If S and T are MQ-schemata we say that a structure A *witnesses the inequivalence of* S and T if and only if either A is a counterexample to the claim that S implies T or A is a counterexample to the claim that T implies S.

1. Let S be the schema

$$(\exists x)(Fx \wedge Gx) \wedge (\exists x)(Fx \wedge \neg Gx) \wedge (\exists x)(\neg Fx \wedge Gx) \wedge (\exists x)(\neg Fx \wedge \neg Gx)$$

   and let T be the schema

$$(\forall x)(Fx \equiv Gx).$$

   (a) (25 points) How many structures with universe of discourse $\{1, 2, 3\}$ are counterexamples to the claim that S implies T?

   (b) (25 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ are counterexamples to the claim that S implies T?

2. (25 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ witness the inequivalence of $(\forall x)(Fx \oplus Gx)$ and $(\forall x)(Fx \equiv Gx)$?

3. (25 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5\}$ witness the inequivalence of $(\exists x)(Fx \wedge Gx)$ and $(\forall x)(Fx \vee Gx)$?