

**LGIC 010 & PHIL 005**  
**Problem Set 2**  
**Spring Term, 2019**  
**DUE IN CLASS MONDAY, FEBRUARY 4**

For the purposes of this problem set, we restrict attention to truth-functional schemata all of whose sentence letters are among  $p_1$  and  $p_2$ , that is, schemata drawn from  $\mathbb{S}(\{p_1, p_2\})$ . We employ the following terminology in these problems.

- We write  $\mathbb{A}(\{p_1, p_2\})$  for the set of truth-assignments to the sentence letters  $p_1, p_2$ .
  - If  $S, T \in \mathbb{S}(\{p_1, p_2\})$ ,  $S$  *implies*  $T$  if and only if for every  $A \in \mathbb{A}(\{p_1, p_2\})$ , if  $A$  satisfies  $S$ , then  $A$  satisfies  $T$ .  $S$  and  $T$  are equivalent if and only if  $S$  implies  $T$  and  $T$  implies  $S$ .
  - A list of truth-functional schemata is *succinct* if and only if no two schemata on the list are equivalent.
  - A list  $L$  of schemata is an *anti-chain* if and only if no schema on  $L$  implies any other schema on  $L$ .
  - An anti-chain  $L$  is *maximal* if and only if for every schema  $S$  not included in  $L$ , the list  $L^*$  obtained from  $L$  by appending  $S$  is not an anti-chain.
  - A truth-functional schema *implies a list of schemata* if and only if it implies every schema on the list.
  - The *power* of a truth-functional schema is the length of a longest succinct list of schemata it implies.
1. (30 points) What is the power of the schema  $p_1 \oplus p_2$ ?
  2. (40 points) Make a list of all the numbers that are lengths of maximal anti-chains. Explain your answer.
  3. (30 points) What is the length of a longest succinct list of schemata all of which imply the schema

$$p_1 \supset p_2.$$