

LGIC 010 & PHIL 005
Problem Set 2
Spring Term, 2018
DUE IN CLASS MONDAY, JANUARY 29

For the purposes of this problem set, we restrict attention to truth-functional schemata all of whose sentence letters are among p_1 and p_2 , that is, schemata drawn from $\mathbb{S}(\{p_1, p_2\})$. We employ the following terminology in these problems.

- We write $\mathbb{A}(\{p_1, p_2\})$ for the set of truth-assignments to the sentence letters p_1, p_2 .
 - If $S, T \in \mathbb{S}(\{p_1, p_2\})$, S *implies* T if and only if for every $A \in \mathbb{A}(\{p_1, p_2\})$, if A satisfies S , then A satisfies T . S and T are equivalent if and only if S implies T and T implies S .
 - A list of truth-functional schemata is *succinct* if and only if no two schemata on the list are equivalent.
 - A list L of schemata is an *anti-chain* if and only if no schema on L implies any other schema on L .
 - An anti-chain L is *maximal* if and only if for every schema S not included in L , the list L^* obtained from L by appending S is not an anti-chain.
 - A truth-functional schema *implies a list of schemata* if and only if it implies every schema on the list.
 - The *power* of a truth-functional schema is the length of a longest succinct list of schemata it implies.
1. (25 points) What is the power of the schema $\neg p_1$?
 2. (25 points) What is the length of a longest anti-chain? Explain your answer.
 3. (25 points) Is there a maximal anti-chain consisting of schemata, each of which is neither valid nor unsatisfiable and none of which have the same power? If so, what are the possible lengths of such maximal anti-chains? Explain your answer.
 4. (25 points) What is the length of a longest succinct list of schemata all of which imply the schema

$$p_1 \oplus p_2.$$