For the purposes of problems 1-3, we restrict attention to truth-functional schemata all of whose sentence letters are among p_1, p_2, p_3, and p_4. We employ the following terminology in these problems.

- A list of truth-functional schemata is *succinct* if and only if no two schemata on the list are equivalent.

- A truth-functional schema *implies a list of schemata* if and only if it implies every schema on the list.

- The *power* of a truth-functional schema is the length of a longest succinct list of schemata it implies.

1. (25 points) What is the power of the schema $(p_1 \oplus p_2) \vee (p_3 \oplus p_4)$?

2. (25 points) Let $S_1, \ldots, S_5$ be a list of five schemata (all of whose sentence letters are among p_1, p_2, p_3, and p_4) satisfying the following condition: for every $1 \leq i < 5$, S_i implies S_{i+1} and S_{i+1} does not imply S_i. What are the maximum and minimum possible values for the power of S_3?

3. (25 points) What is the length of a longest succinct list of schemata (all of whose sentence letters are among p_1, p_2, p_3, and p_4) each of which implies no schema of power 256?

4. (25 points) Is the conjunction of the following schemata truth-functionally satisfiable? Explain your answer.

   - $(p_{12} \equiv p_{21}) \wedge (p_{13} \equiv p_{31}) \wedge (p_{14} \equiv p_{41}) \wedge (p_{23} \equiv p_{32}) \wedge (p_{24} \equiv p_{42}) \wedge (p_{34} \equiv p_{43})$
   - $(p_{12} \oplus p_{13}) \oplus p_{14}$
   - $(p_{21} \oplus p_{23}) \oplus p_{24}$
   - $(p_{31} \oplus p_{32}) \oplus p_{34}$
   - $\neg((p_{41} \oplus p_{42}) \oplus p_{43})$