1. (25 points) We say a set of integers is *good* if and only if it does not contain a pair of distinct integers whose difference is divisible by 20. What is the largest number n such that there is a good set with n members? Give an example of a good set of that size. Explain why there is no larger good set.

2. (25 points) Dojo master Takeda organized a day of one-on-one combat in which all 28 trainees at the martial arts academy faced at least one opponent. At the end of the day, Takeda interrogated all but one of the trainees, Jules, and learned that no two of those 27 had faced the same number of opponents. How many different opponents did Jules face? Explain your answer.

3. (25 points) How many truth assignments to the sentence letters $p_1, \ldots, p_{10}$ satisfy the following schema?

$$(p_1 \supset p_2) \wedge (p_2 \supset p_3) \wedge \ldots \wedge (p_8 \supset p_9) \wedge (p_9 \supset p_{10})$$

4. (25 points) Recall that $\oplus$ represents exclusive disjunction. How many truth assignments to the sentence letters $p_1, \ldots, p_5, q_1, \ldots, q_5$ satisfy the following schema?

$$(p_1 \oplus q_1) \wedge (p_2 \oplus q_2) \wedge (p_3 \oplus q_3) \wedge (p_4 \oplus q_4) \wedge (p_5 \oplus q_5)$$